

Lecture 23:

Domain-Specific Programming Systems

**Parallel Computer Architecture and Programming
CMU 15-418, Spring 2013**

Slide acknowledgments:

Pat Hanrahan, Zach Devito (Stanford University)

Jonathan Ragan-Kelley (MIT)

Course themes:

Designing computer systems that scale

(running faster given more resources)

Designing computer systems that are efficient

(running faster under constraints on resources)

Techniques discussed:

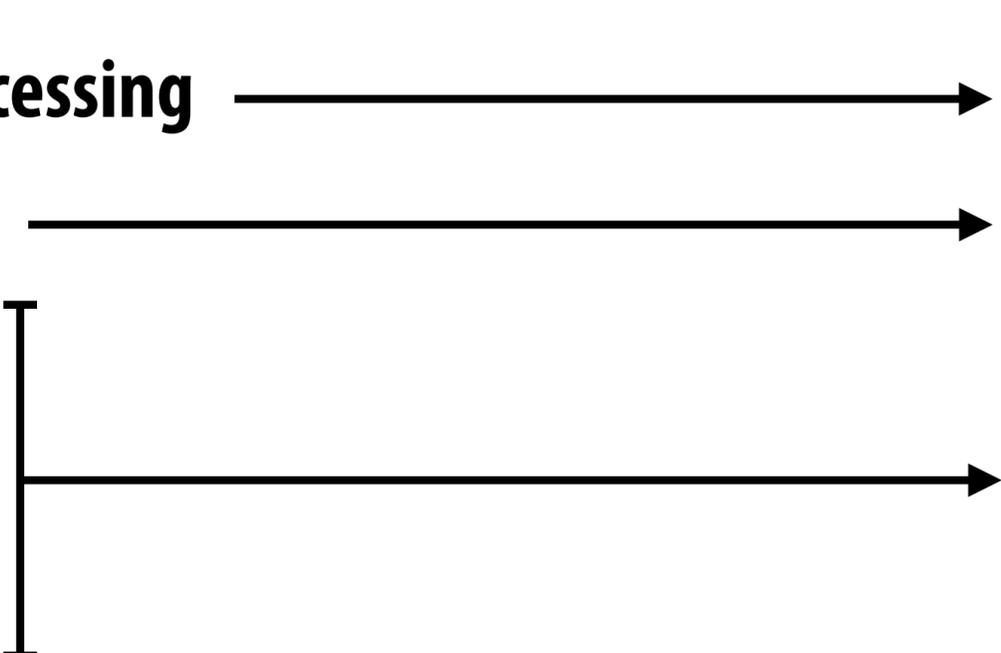
Exploiting parallelism in applications

Exploiting locality in applications

Leveraging HW specialization

Hardware trend: specialization of execution

■ Multiple forms of parallelism

- SIMD/vector processing → Fine-granularity parallelism: similar execution on different data
 - Multi-threading → Mitigate inefficiencies of unpredictable data access
 - Multi-core
 - Multiple node → Varying scales of coarse-granularity parallelism
 - Multiple server
- 

■ Heterogeneous execution capability

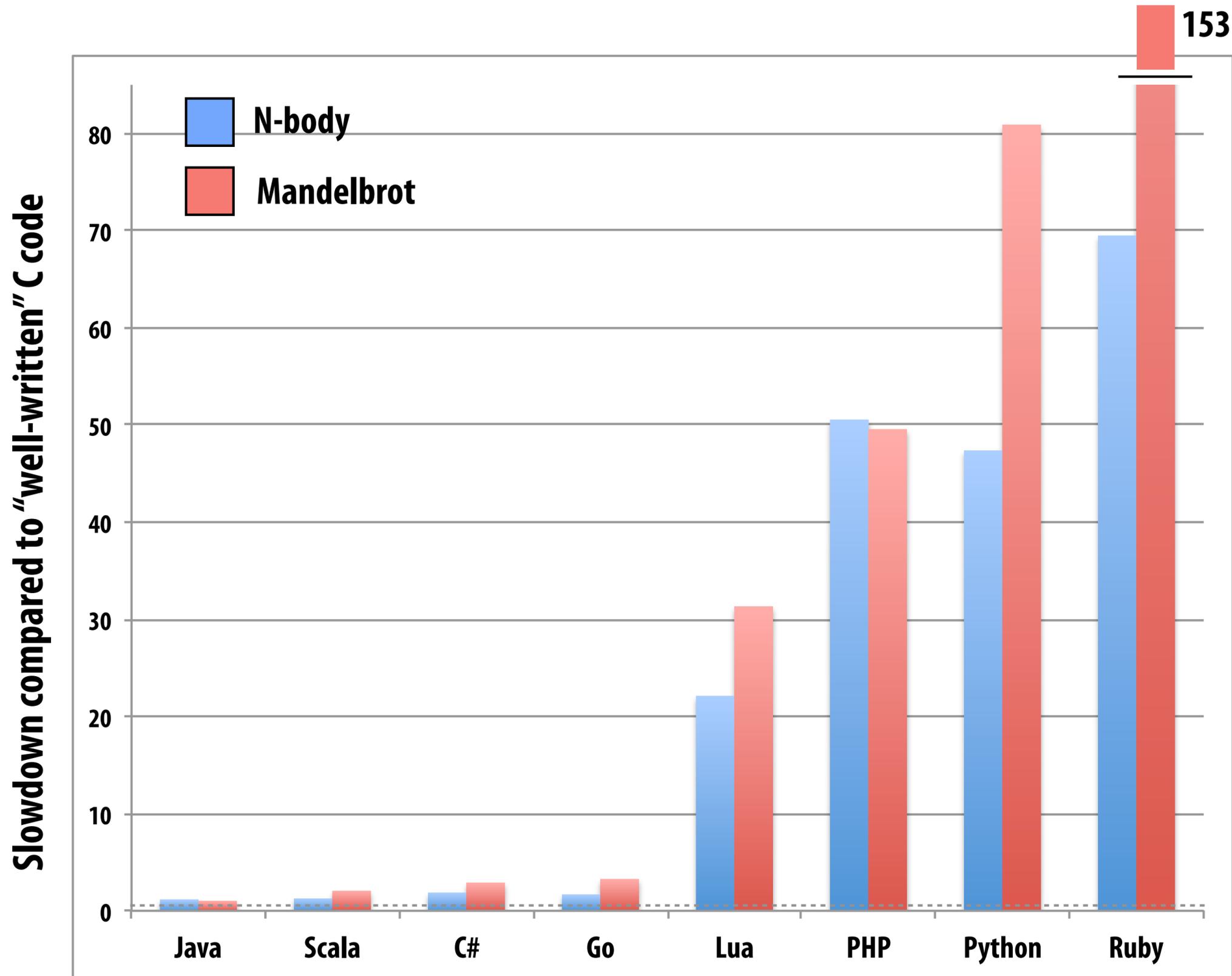
- Programmable, latency-centric (e.g., “CPU-like” cores)
- Programmable, throughput-optimized (e.g., “GPU-like” cores)
- Fixed-function, application-specific (e.g., image/video/audio processing)

Motivation for parallelism and specialization: maximize compute capability given constraints on chip area, power

Most software uses modern HW resources inefficiently

- **Consider basic sequential C code (baseline performance)**
 - **Well-written sequential C code: ~ 5-10x faster**
 - **Assembly language program: another small constant factor faster**
 - **Java, Python, PHP, etc. ??**

Code performance relative to C (single core)



Source: The Computer Language Benchmarks Game: <http://shootout.alioth.debian.org>

Even good C code is inefficient

Recall Assignment 1's Mandelbrot program

For execution on this laptop: quad-core, Intel Core i7, AVX instructions...

Single core, with AVX vector instructions: 5.8x speedup over C implementation

Multi-core + hyper-threading + vector instructions: 21.7x speedup

Conclusion: basic C implementation leaves a lot of performance on the table

Making efficient use of modern machines is challenging (proof by assignments 2, 3, and 4)

**In our assignments you only programmed homogeneous parallel computers.
And parallelism in that context was not easy.**

Assignment 2: GPU cores only

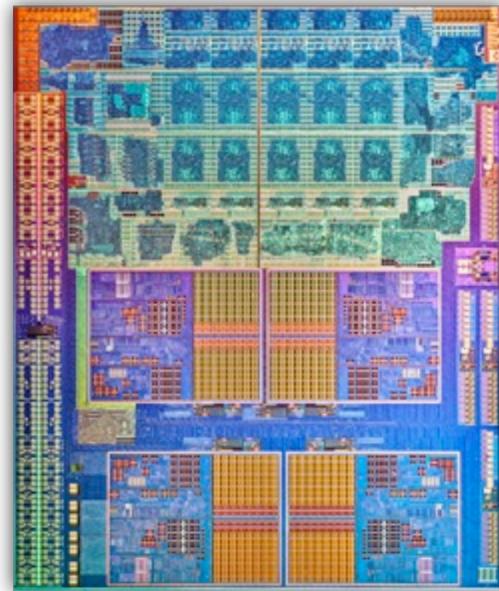
Assignment 3: Blacklight (multiple CPUs with relatively fast interconnect)

Interesting: no one attempted to utilize SIMD execution on assignment 3 to further improve performance

Assignment 4 : multiple parallel machines

Power-efficient heterogeneous platforms

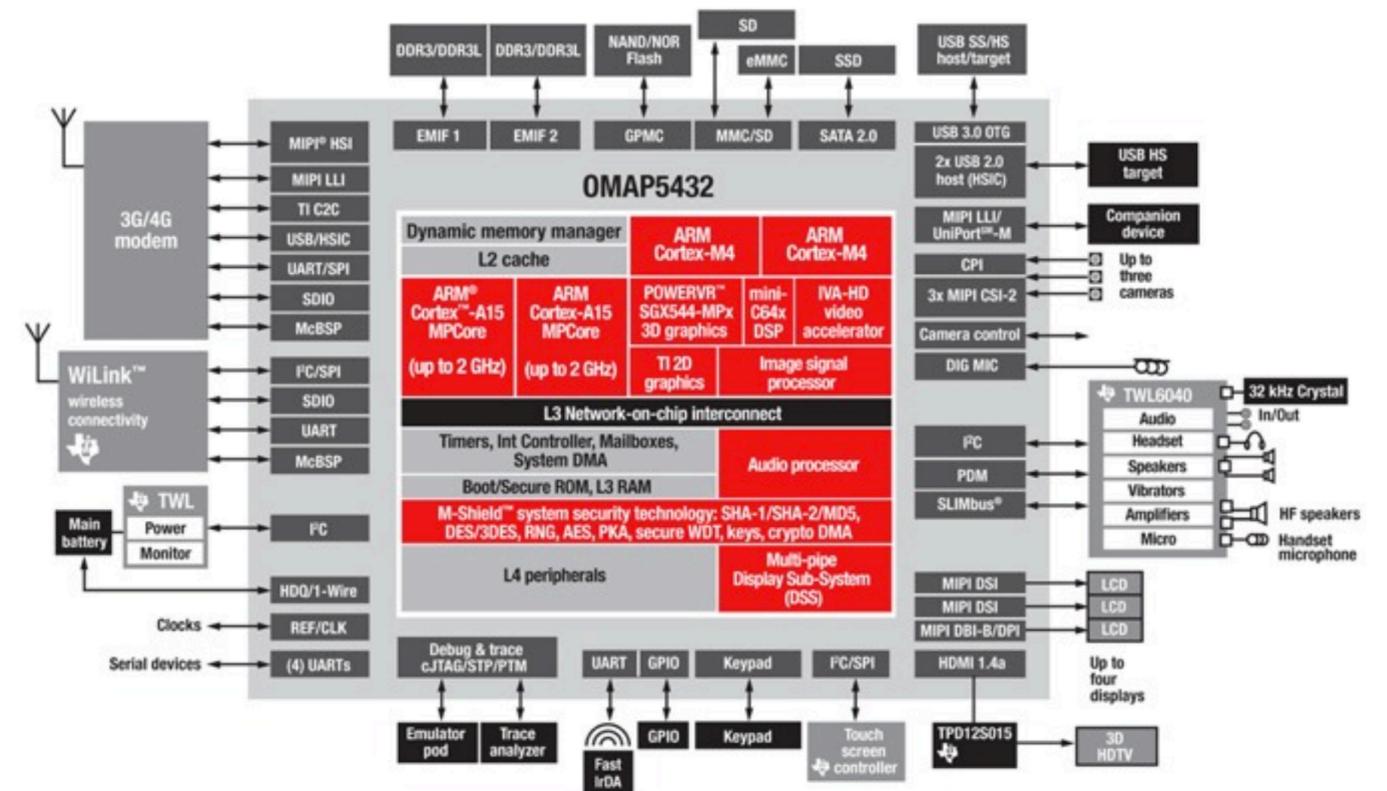
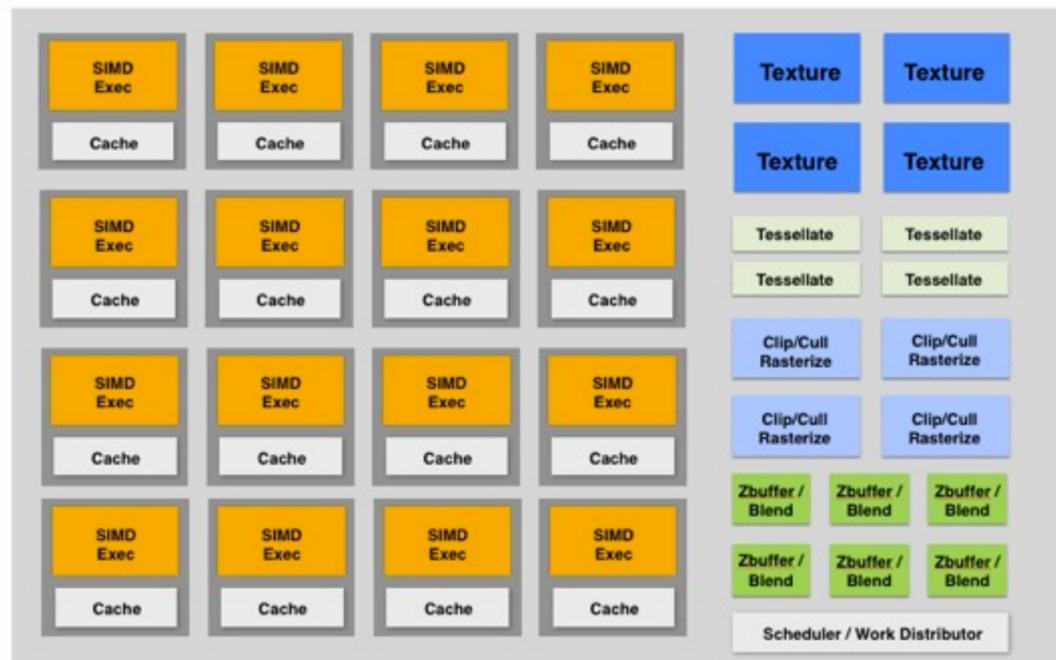
**Integrated
CPU + GPU**



CPU+data-parallel accelerator



**GPU:
throughput cores + fixed-function**



**Mobile system-on-a-chip:
CPU+GPU+media processing**

Huge challenge

- **Machines with very different performance characteristics**
- **Worse: different technologies and performance characteristics within the same machine at different scales**
 - **Within a core: SIMD, multi-threading: fine-granularity sync and comm.**
 - **Across cores: coherent shared memory via fast on-chip network**
 - **Hybrid CPU+GPU multi-core: incoherent (potentially) shared memory**
 - **Across racks: distributed memory, multi-stage network**

Variety of programming models to abstract HW

- **Machines with very different performance characteristics**
- **Worse: different technologies and performance characteristics within the same machine at different scales**
 - **Within a core: SIMD, multi-threading: fine grained sync and comm.**
 - **Abstractions: SPMD programming (ISPC, Cuda, OpenCL)**
 - **Across cores: coherent shared memory via fast on-chip network**
 - **Abstractions: OpenMP shared address space, Cilk, TBB**
 - **Hybrid CPU+GPU multi-core: incoherent (potentially) shared memory**
 - **Abstractions: OpenCL**
 - **Across racks: distributed memory, multi-stage network**
 - **Abstractions: message passing (MPI, Go channels, Charm++)**

Huge challenge

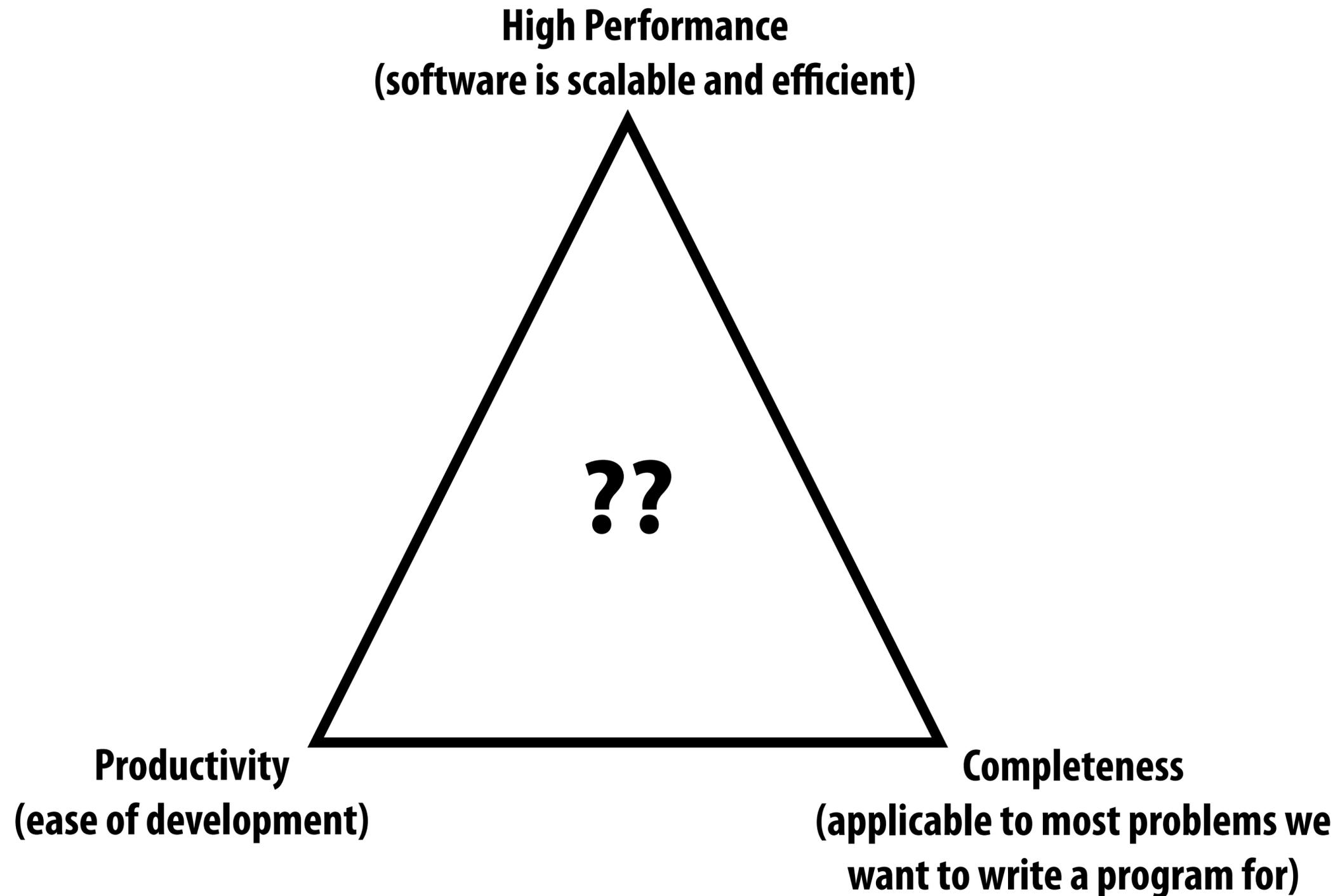
- **Machines with very different performance characteristics**
- **Worse: different performance characteristics within the same machine at different scales**
- **To be efficient, software must be optimized for HW characteristics**
 - **Difficult even in the case of one level of one machine ****
 - **Combinatorial complexity of optimizations when considering a complex machine, or different machines**
 - **Loss of software portability**

**** Little success developing automatic tools to identify efficient HW mapping for arbitrary, complex applications**

Open compute science question:

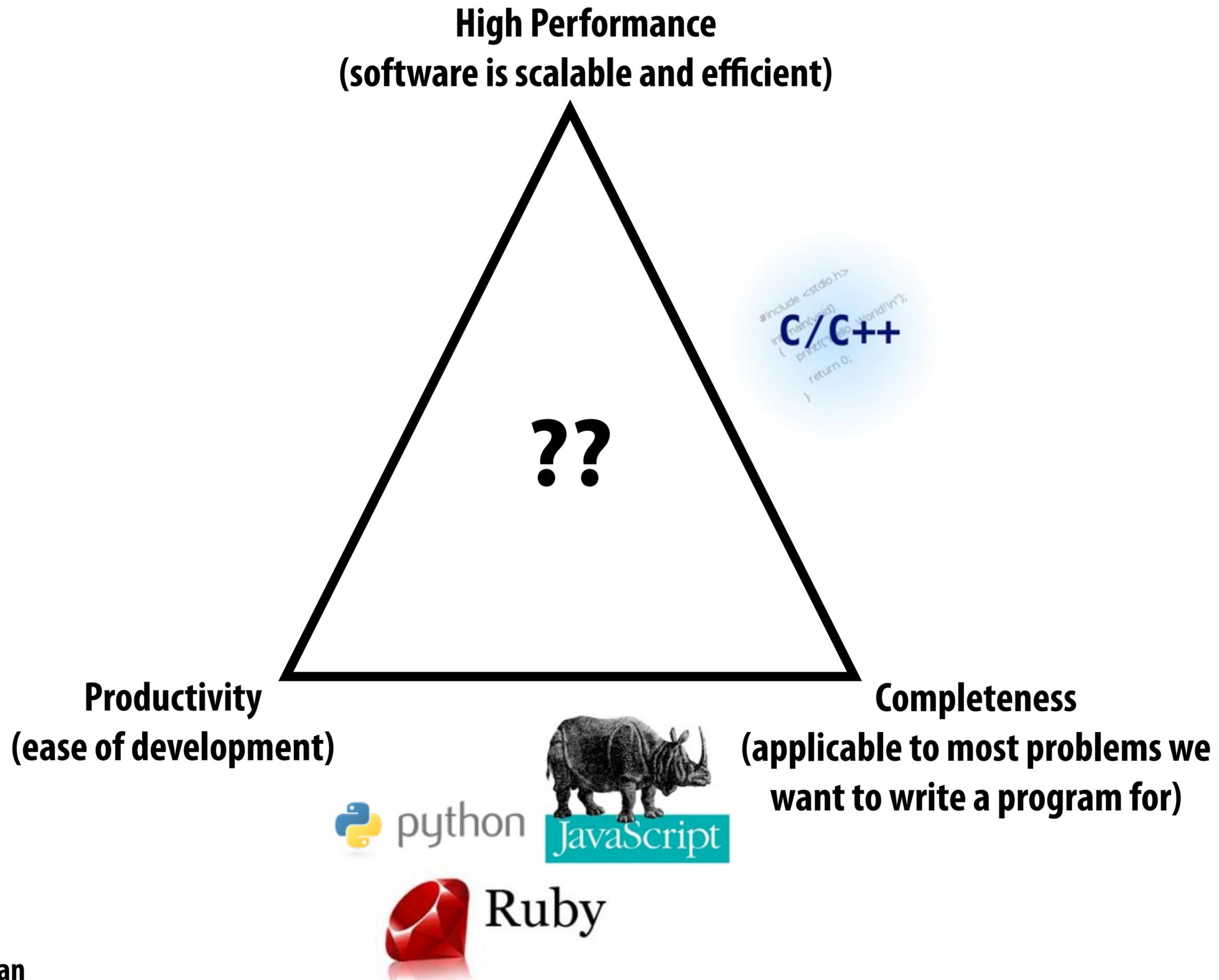
How do we enable programmers to write software that efficiently uses these parallel machines?

The [magical] ideal parallel programming language



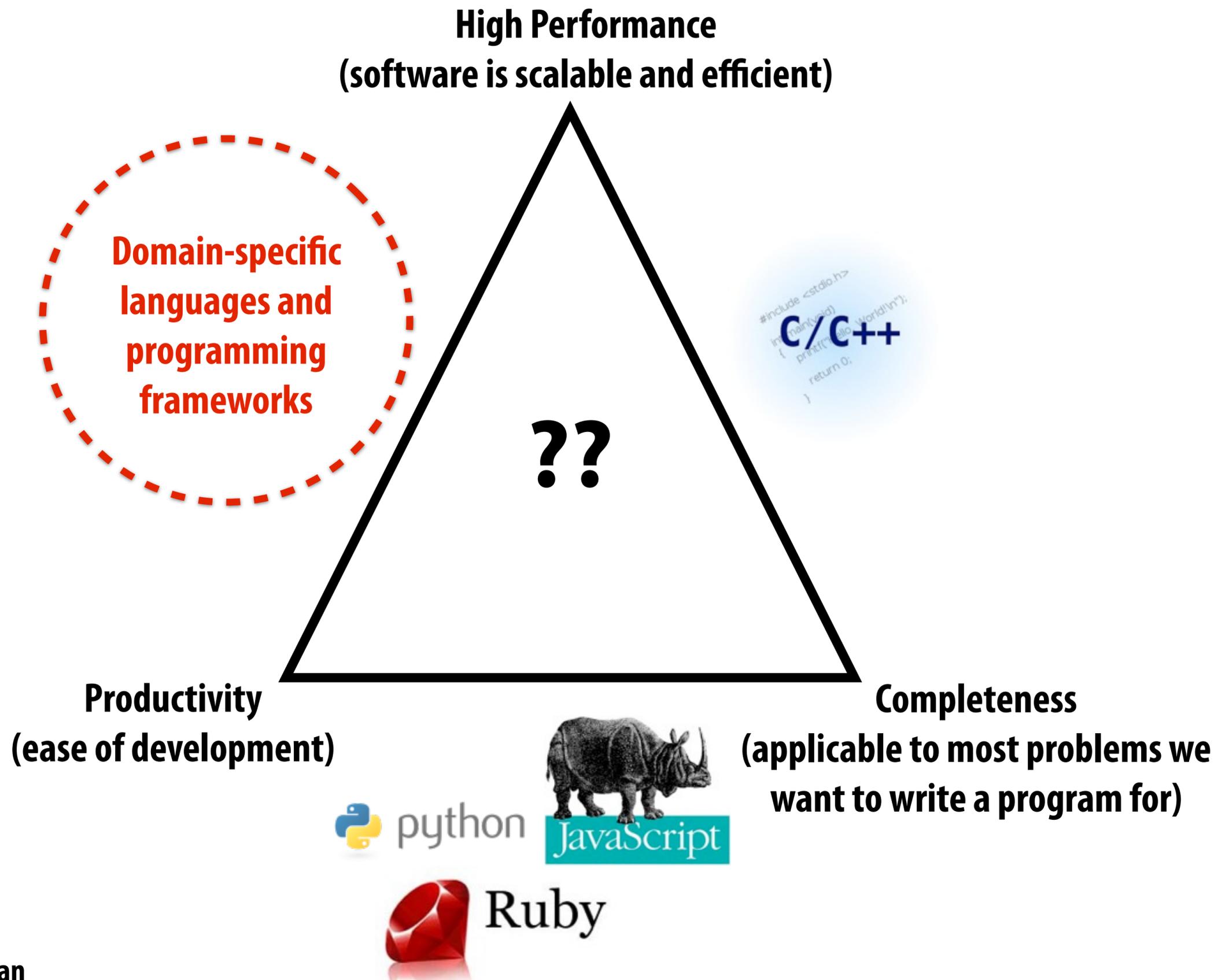
Successful programming languages

Here: definition of success = widely used



Growing interest in domain-specific programming systems

To realize high performance and productivity: willing to sacrifice completeness



Domain-specific programming systems

- **Main idea: raise level of abstraction**
- **Introduce high-level programming primitives specific to domain**
 - **Productive: intuitive to use, portable across machines, primitives correspond to behaviors frequently used to solve problems in domain**
 - **Performant: system uses domain knowledge to provide efficient, optimized implementation(s)**
 - **Given a machine: system knows what algorithms to use, parallelization strategies to employ for this domain**
 - **Optimization goes beyond efficient mapping of software to hardware! The hardware platform itself can be optimized to the abstractions as well**
- **Cost: loss of generality/completeness**

Two domain-specific programming examples

1. Liszt: Scientific computing

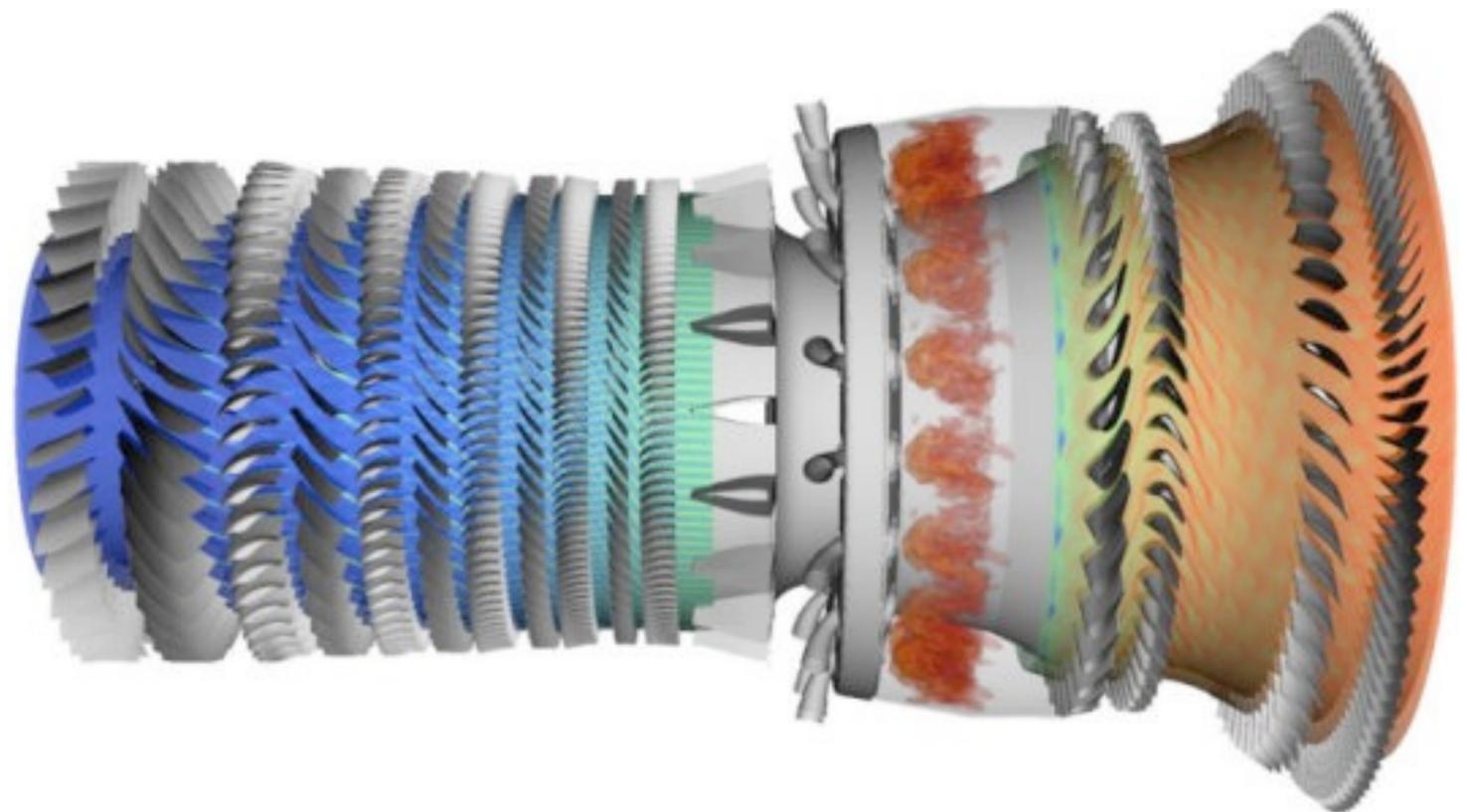
2. Halide: Image processing

(Bonus slides contain third example: OpenGL)

DSL Example 1:

Lizst: a language for solving PDE's on meshes

[DeVito et al. Supercomputing 11, SciDac '11]



Slide credit for this section of lecture:
Pat Hanrahan and Zach DeVito (Stanford)

<http://lizst.stanford.edu/>

Fields on unstructured meshes

Fields

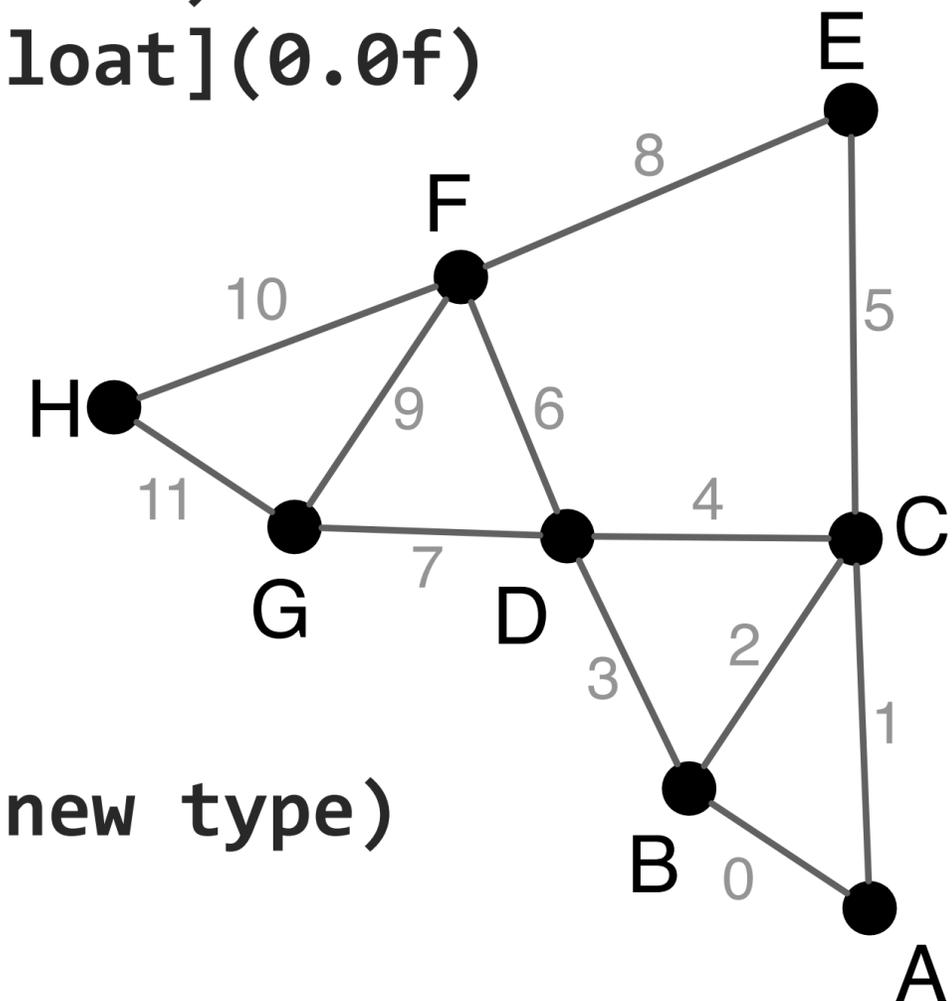
Mesh Entity

```
val Position = FieldWithLabel[Vertex, Float3]("position")
```

```
val Temperature = FieldWithConst[Vertex, Float](0.0f)
```

```
val Flux = FieldWithConst[Vertex, Float](0.0f)
```

```
val JacobiStep = FieldWithConst[Vertex, Float](0.0f)
```



Notes:

Fields are a higher-kinded type

(special function that maps a type to a new type)

Explicit algorithm: heat conduction on grid

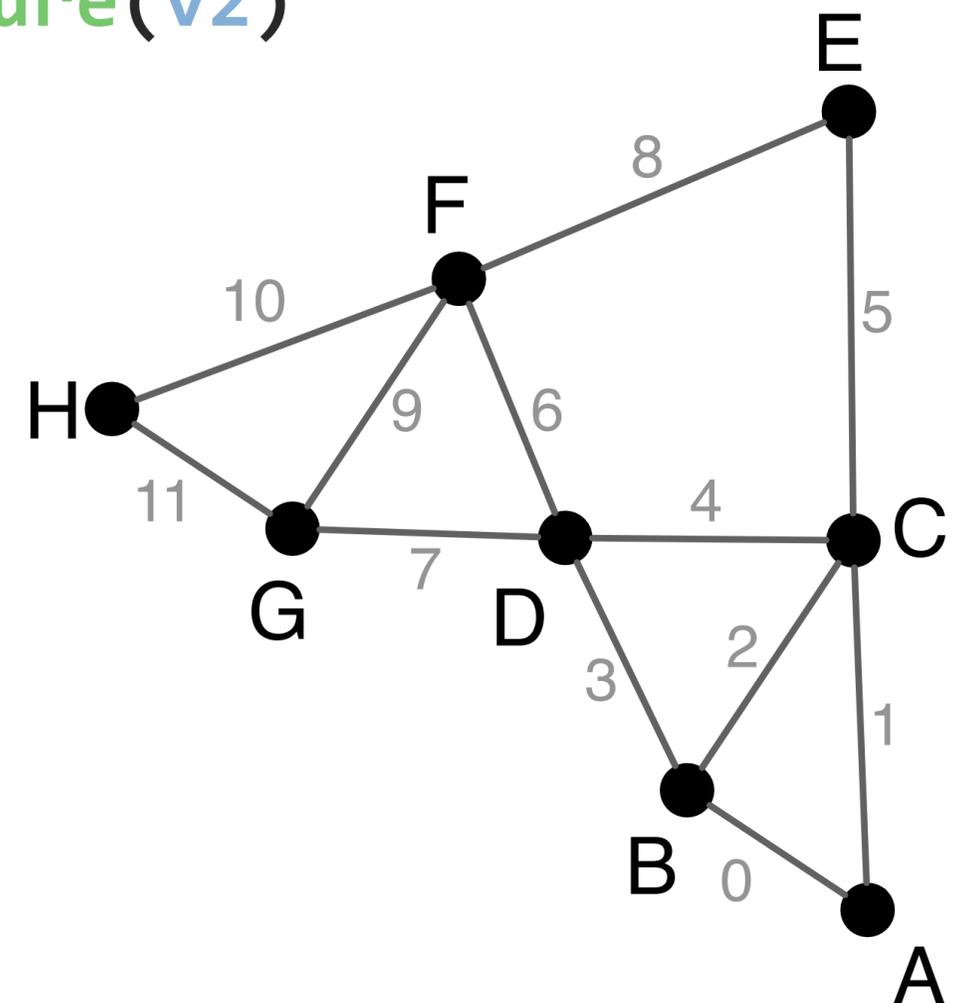
```
var i = 0;
while ( i < 1000 ) {
  Flux(vertices(mesh)) = 0.f;
  JacobiStep(vertices(mesh)) = 0.f;
  for (e <- edges(mesh)) {
    val v1 = head(e)
    val v2 = tail(e)
    val dP = Position(v1) - Position(v2)
    val dT = Temperature(v1) - Temperature(v2)
    val step = 1.0f/(length(dP))
    Flux(v1) += dT*step
    Flux(v2) -= dT*step
    JacobiStep(v1) += step
    JacobiStep(v2) += step
  }
  i += 1
}
```

Fields

Mesh

Topology Functions

Iteration over Set

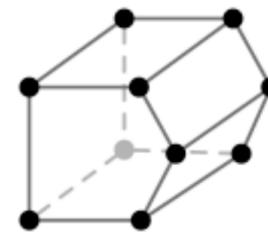


Liszt topological operators



```
BoundarySet1[ME <: MeshElement](name : String) : Set[ME]
vertices(e : Mesh) : Set[Vertex]
cells(e : Mesh) : Set[Cell]
edges(e : Mesh) : Set[Edge]
faces(e : Mesh) : Set[Face]
```

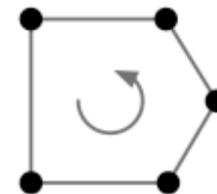
- ```
vertices(e : Vertex) : Set[Vertex]
cells(e : Vertex) : Set[Cell]
edges(e : Vertex) : Set[Edge]
faces(e : Vertex) : Set[Face]
```



```
cells(e : Cell) : Set[Cell]
vertices(e : Cell) : Set[Vertex]
faces(e : Cell) : Set[Face]
edges(e : Cell) : Set[Edge]
```

- 

```
vertices(e : Edge) : Set[Vertex]
facesCCW2(e : Edge) : Set[Face]
cells(e : Edge) : Set[Cell]
head(e : Edge) : Vertex
tail(e : Edge) : Vertex
flip4(e : Edge) : Edge
towards5(e : Edge, t : Vertex) : Edge
```



```
cells(e : Face) : Set[Cell]
edgesCCW2(e : Face) : Set[Edge]
vertices(e : Face) : Set[Vertex]
inside3(e : Face) : Cell
outside3(e : Face) : Cell
flip4(e : Face) : Face
towards5(e : Face, t : Cell) : Face
```

# Liszt programming

- **Liszt program describes operations on fields of an abstract mesh representation**
- **Application specifies type of mesh (regular, irregular) and its topology**
- **Mesh representation is chosen by Liszt**
  - **Based on mesh type, program behavior, and machine**

# Compiling to parallel computers

Recall challenges you have faced in your assignments

- 1. Identify parallelism**
- 2. Identify data locality**
- 3. Reason about required synchronization**

# Key: determining program dependencies

## 1. Identify parallelism

- Absence of dependencies implies can be executed in parallel

## 2. Identify data locality

- Partition data based on dependencies (localize dependent computations for faster synchronization)

## 3. Reason about required synchronization

- Synchronization is needed to respect existing dependencies (must wait until the values a computation depends on are known)

In general programs, compilers are unable to infer dependencies at global scale:  $a[i] = b[f(i)]$  (must execute  $f(i)$  to know dependency)

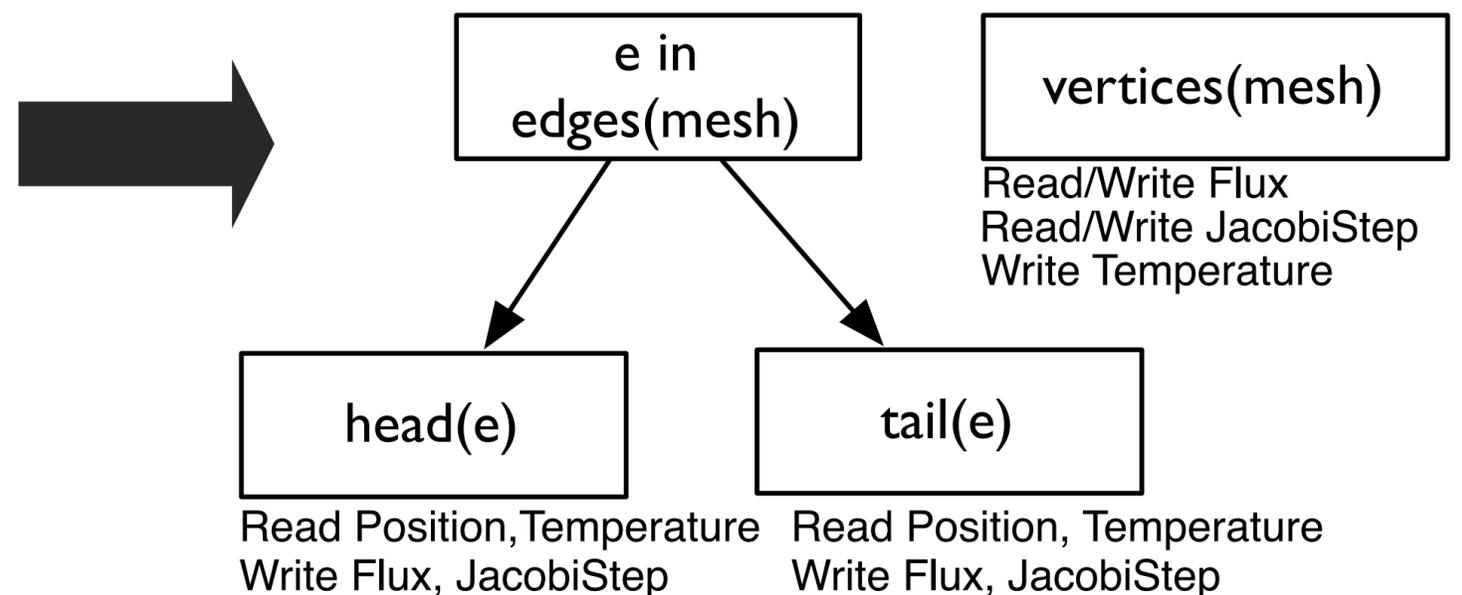
# Liszt is constrained to allow dependency analysis

**Inferring stencils:** (“stencil” = mesh elements accessed in iteration of loop = dependencies for the iteration)

Statically analyze code to find stencil of each top-level **for** loop

- Extract nested mesh element reads
- Extract field operations

```
for (e <- edges(mesh)) {
 val v1 = head(e)
 val v2 = tail(e)
 val dP = Position(v1) - Position(v2)
 val dT = Temperature(v1) - Temperature(v2)
 val step = 1.0f/(length(dP))
 Flux(v1) += dT*step
 Flux(v2) -= dT*step
 JacobiStep(v1) += step
 JacobiStep(v2) += step
}
...
```



# Restrict language for dependency analysis

## “Language Restrictions”

- Mesh elements only accessed through built-in topological functions:

```
cells(mesh), ...
```

- Single static assignment:

```
val v1 = head(e)
```

- Data in Fields can only be accessed using mesh elements:

```
Pressure(v)
```

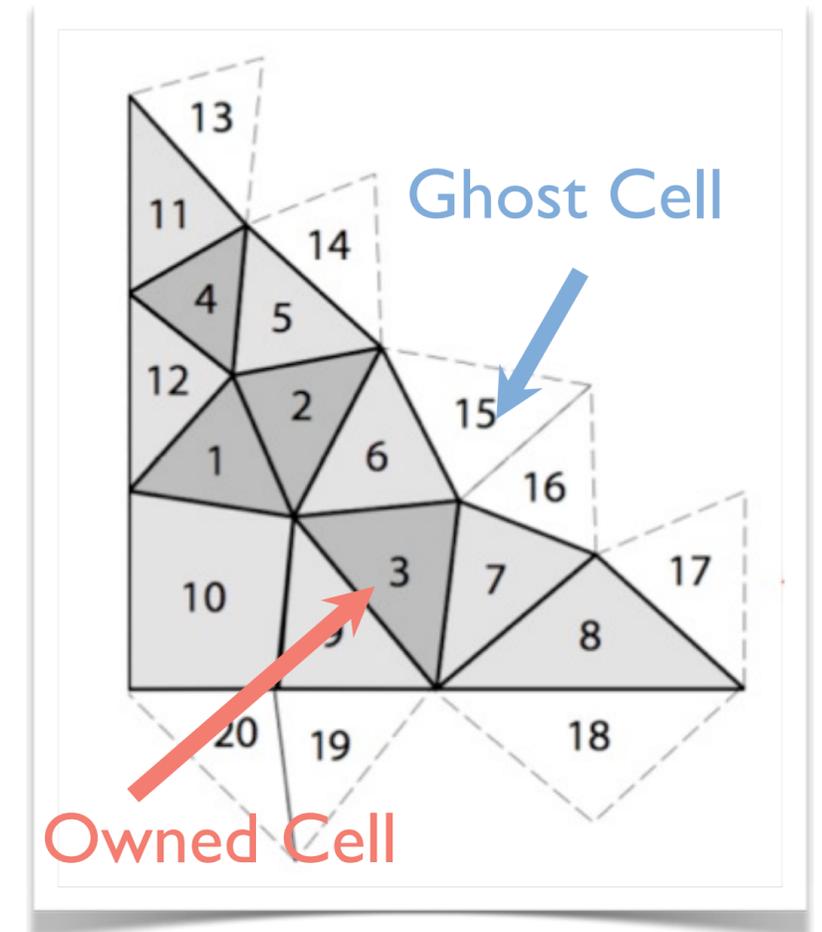
- No recursive functions

**Allows compiler to automatically infer stencil**

# Portable parallelism: use dependencies to implement different parallel execution strategies

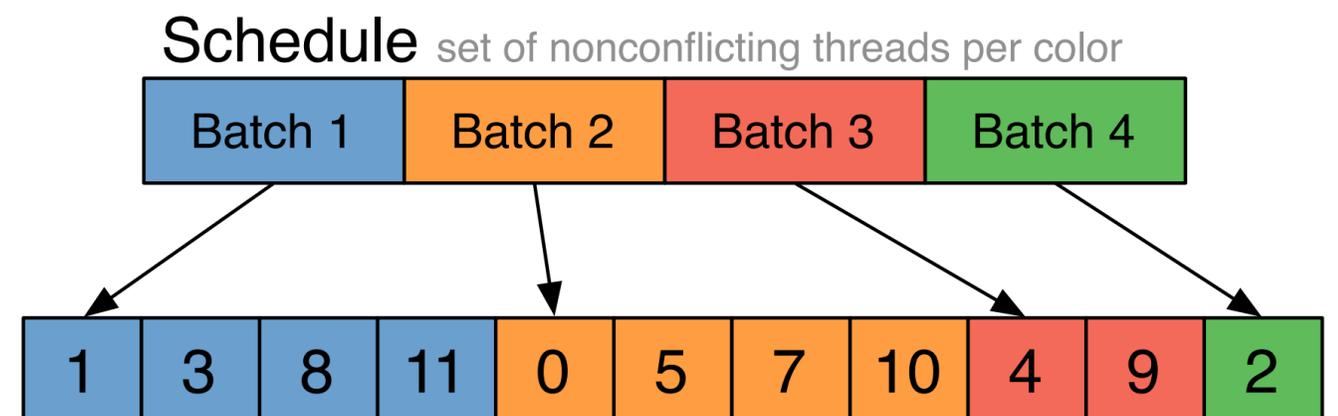
## Partitioning

- Assign partition to each computational unit
- Use **ghost** elements to coordinate boundary communication.



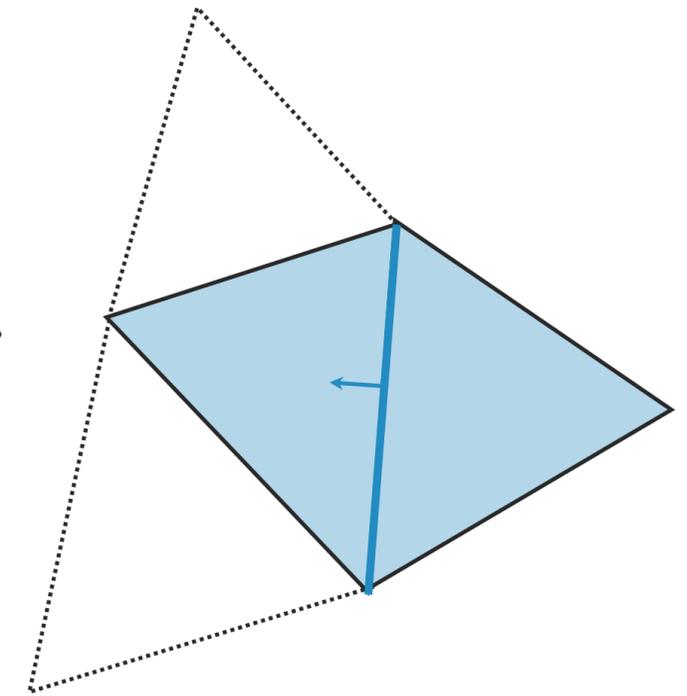
## Coloring

- Calculate interference between work items on domain
- Schedule work-items into non-interfering batches

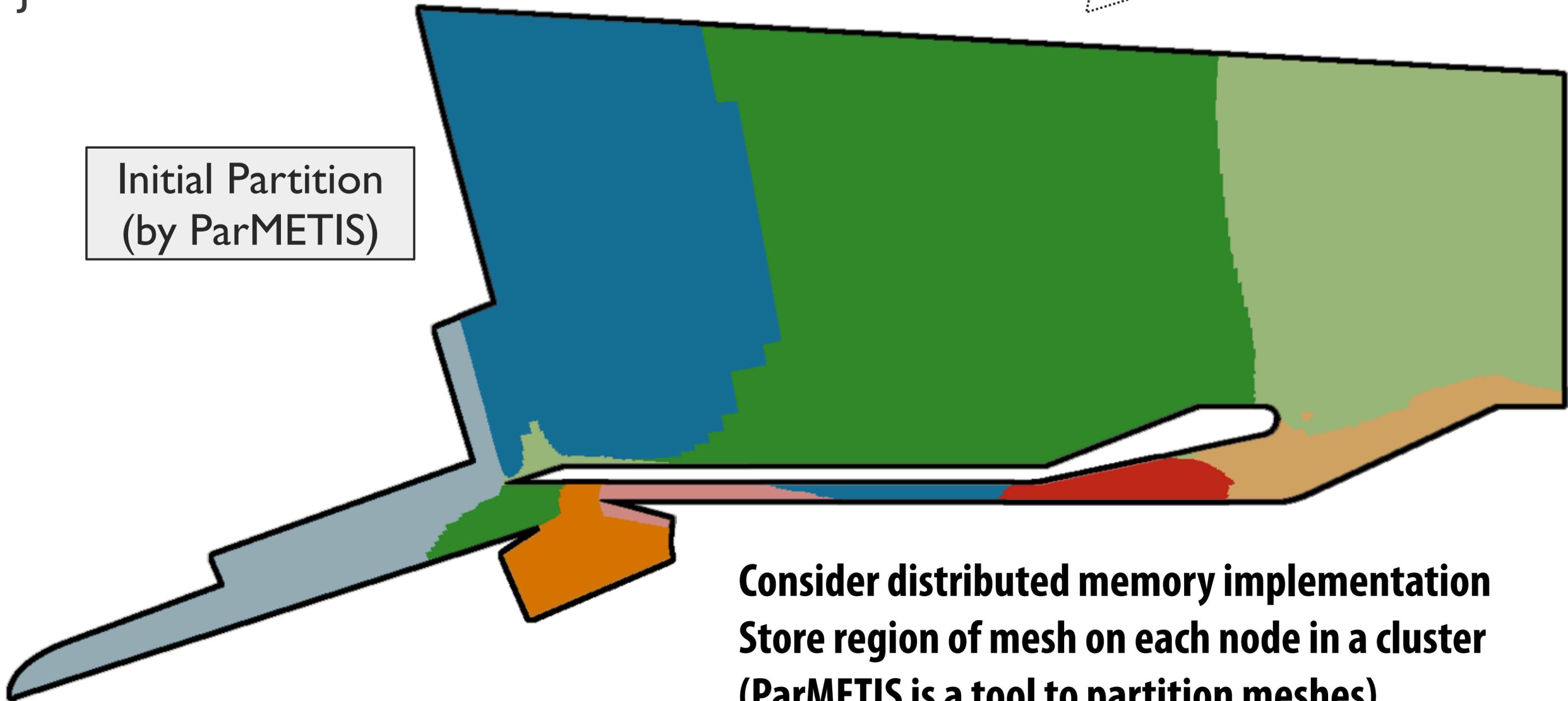


# Distribution memory implementation: Mesh + Stencil -> Graph -> Partition

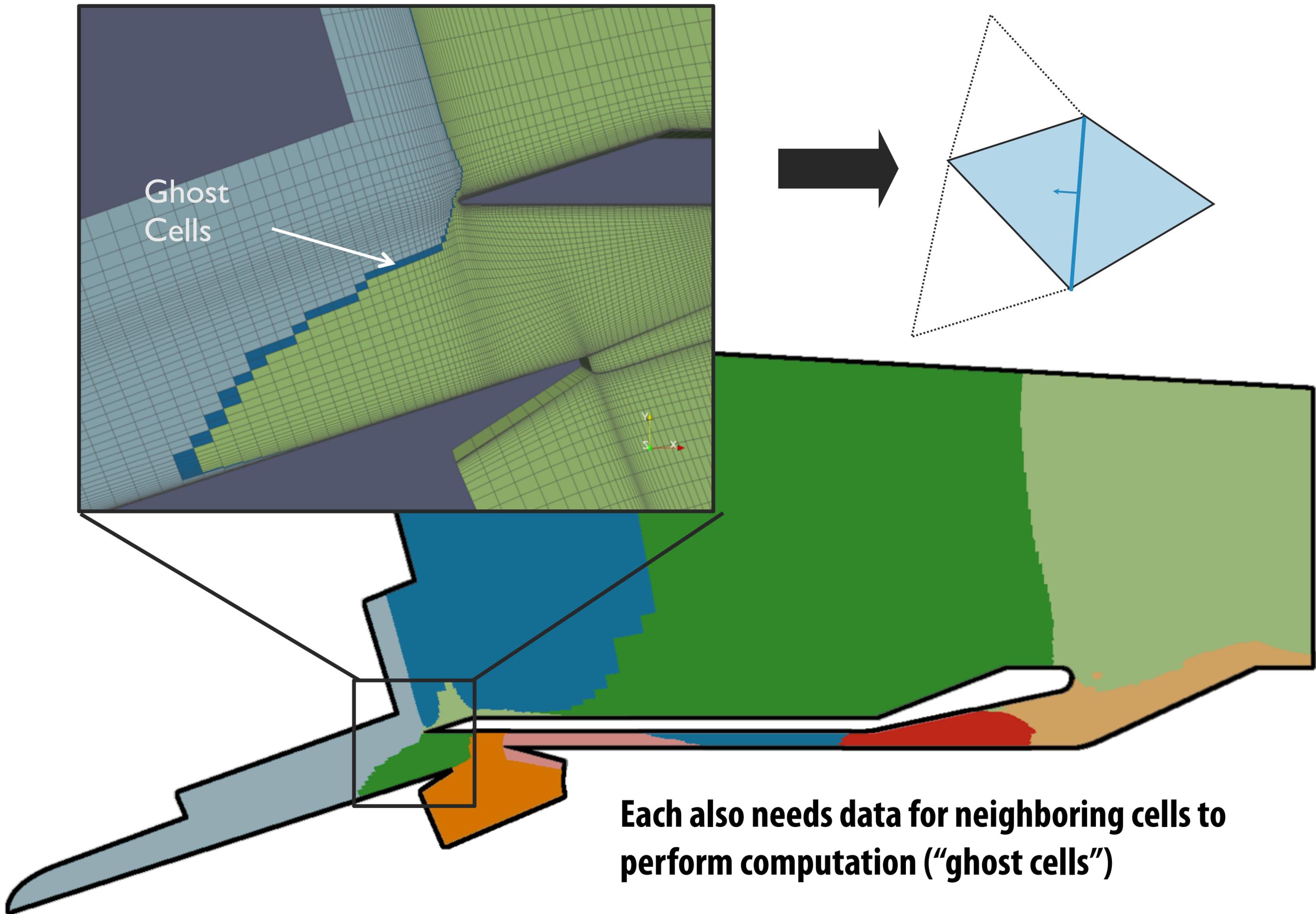
```
for(f <- faces(mesh)) {
 rhoOutside(f) :=
 calc_flux(f, rho(outside(f)))
 + calc_flux(f, rho(inside(f)))
}
```



Initial Partition  
(by ParMETIS)



**Consider distributed memory implementation**  
**Store region of mesh on each node in a cluster**  
**(ParMETIS is a tool to partition meshes)**

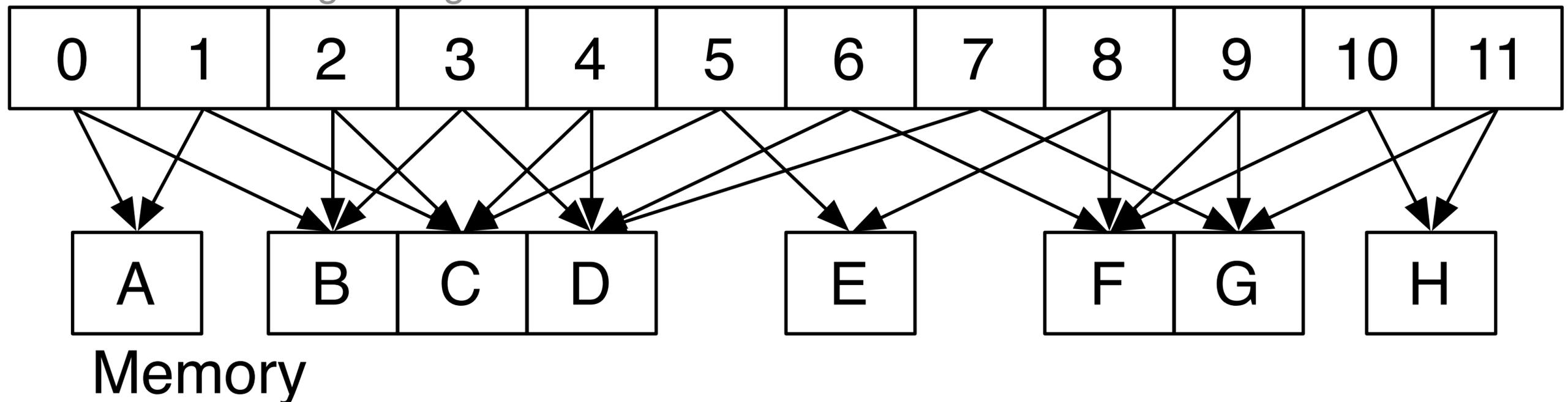


# GPU implementation: parallel reductions

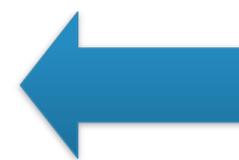
Previous example, one region of mesh per processor (or node in MPI cluster)

On GPU, natural parallelization is one edge per CUDA thread

Threads 1 edge assigned to 1 thread



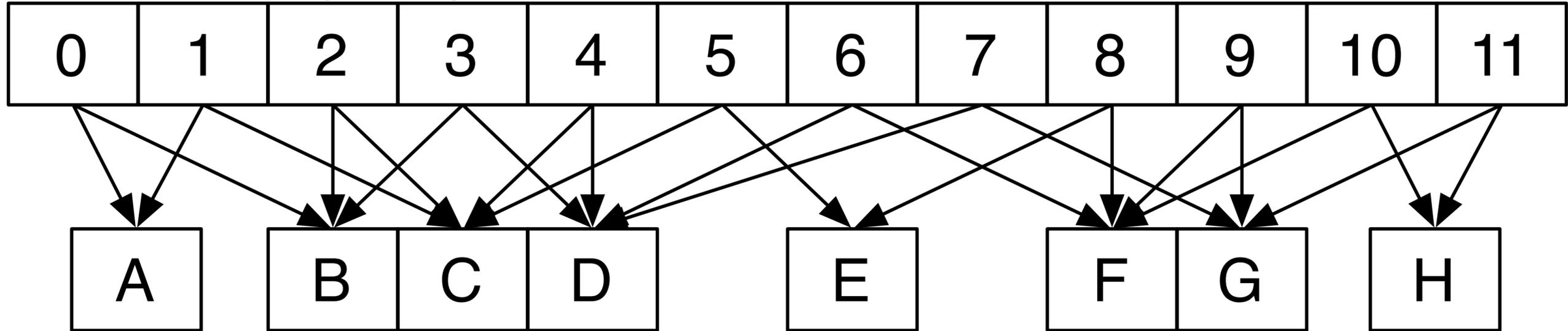
```
for (e <- edges(mesh)) {
 ...
 Flux(v1) += dT*step
 Flux(v2) -= dT*step
 ...
}
```



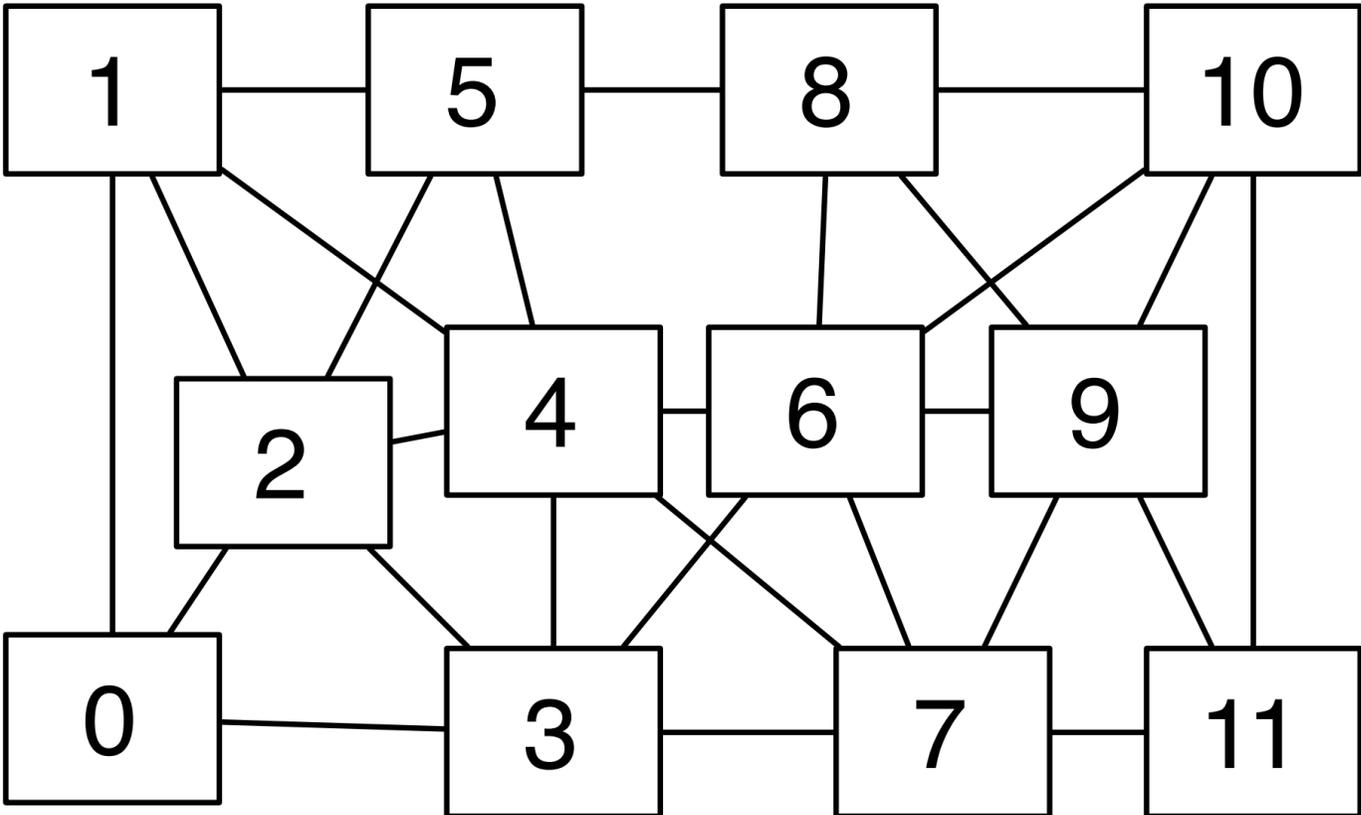
**Different edges share a vertex: requires atomic update of per-vertex field data**

# GPU implementation: conflict graph

Threads 1 edge assigned to 1 thread



Memory

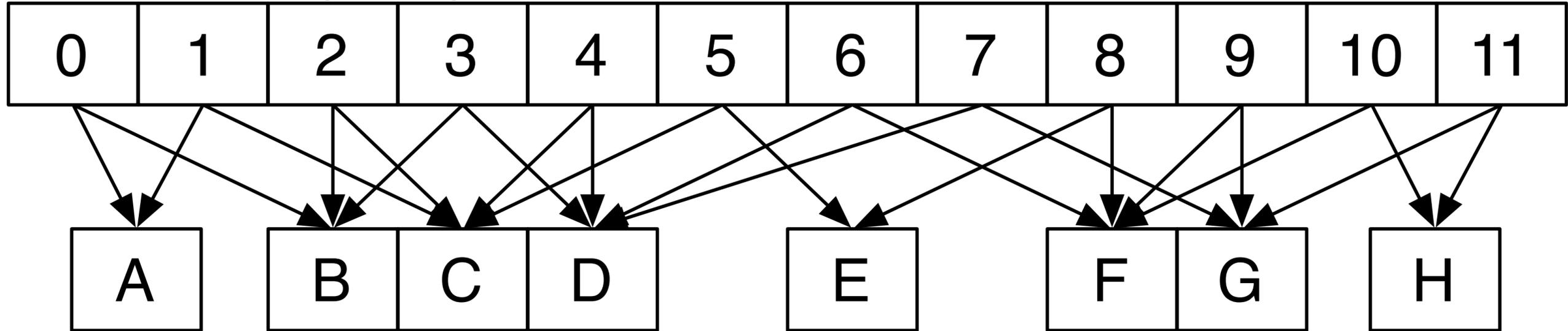


**Identify mesh edges with colliding writes  
(lines in graph indicate presence of collision)**

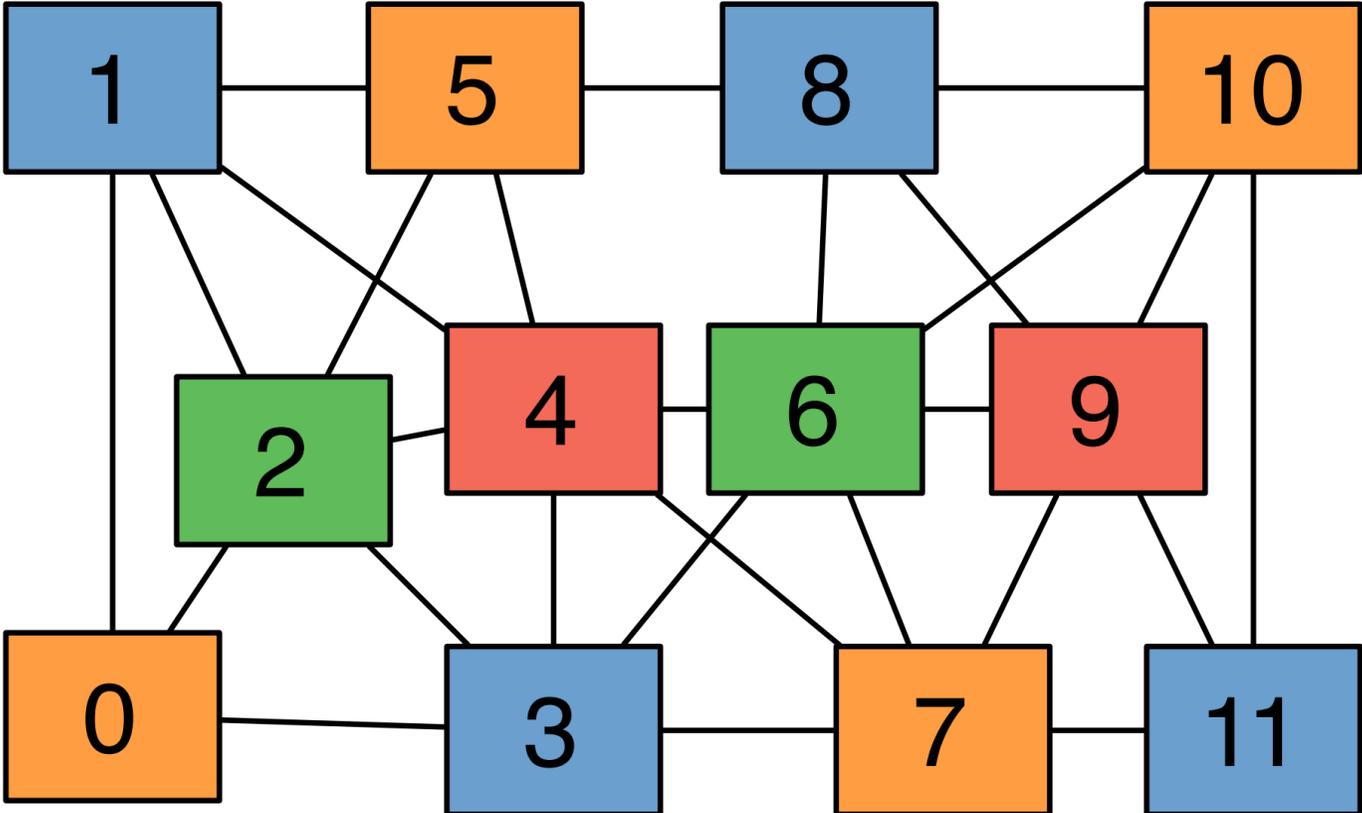
**Can run program to get this information.  
(results valid provided mesh does not change)**

# GPU implementation: conflict graph

Threads 1 edge assigned to 1 thread



Memory

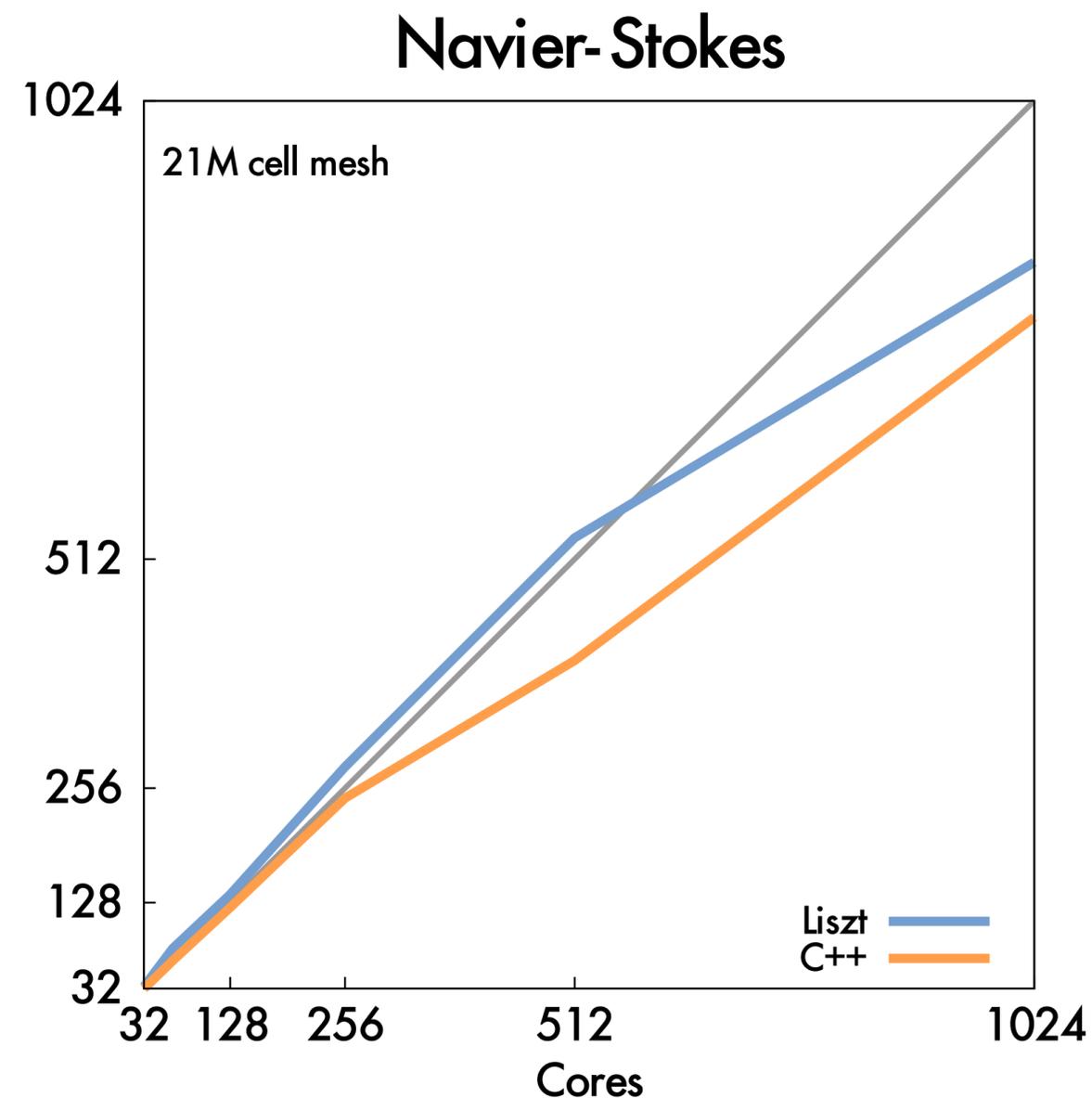
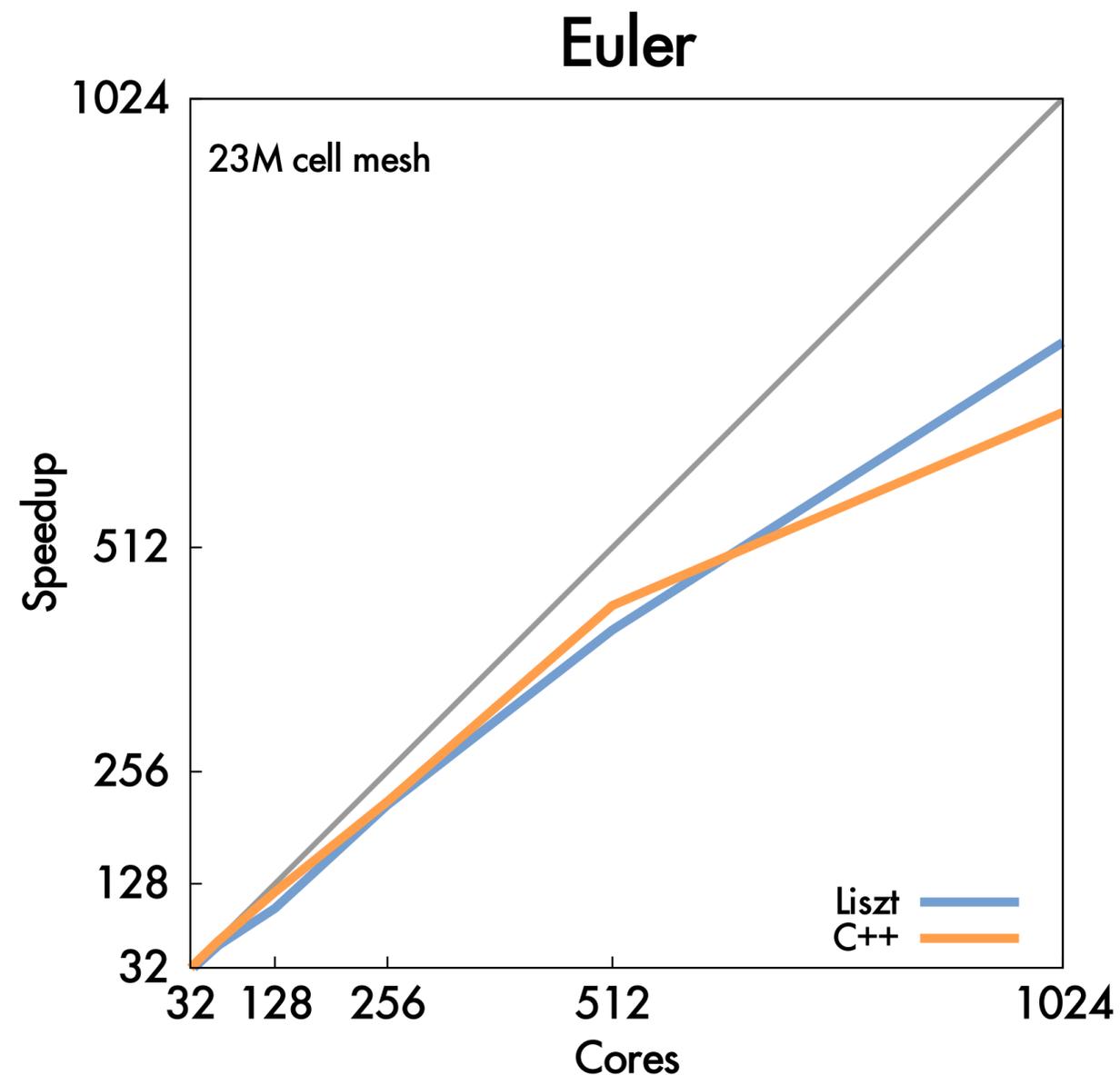


“Color” nodes in graph such that no connected nodes have the same color

Can execute on GPU in parallel, without atomic operations, by running all nodes with the same color in a single CUDA launch.

# MPI Performance

256 nodes, 8 cores per node



**Important:**

**Performance portability!**

**Same Liszt program also runs with high efficiency on GPU (results not shown here).**

# Liszt summary

## ■ Productivity:

- Abstract representation of mesh: vertices, edges, faces, fields
- Intuitive topological operators

## ■ Portability

- Same code runs on cluster of CPUs (MPI runtime) and GPUs

## ■ High-Performance

- Language constrained to allow compiler to track dependencies
- Used for locality-aware partitioning in distributed memory implementation
- Used for graph coloring in GPU implementation
- Completely different parallelization strategies for different platforms
- Underlying mesh representation customized based on usage and platform (e.g, struct of arrays vs. array of structs)

# **DSL Example 2:**

## **Halide: a DSL for image processing**

**Slide acknowledgments:**  
**Jonathan Ragan-Kelley (MIT)**

# What does this C++ code do?

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
 for (int x = 0; x < in.width(); x++)
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;

 for (int y = 0; y < in.height(); y++)
 for (int x = 0; x < in.width(); x++)
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

# Image blur in C++

3x3 convolution (work efficient, two-pass implementation)

~ 9.9 ms per pixel on a modern CPU

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
 for (int x = 0; x < in.width(); x++)
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;

 for (int y = 0; y < in.height(); y++)
 for (int x = 0; x < in.width(); x++)
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

# Optimized C++ code: 3x3 image blur

Good: 10x faster: ~ 0.9 ms per pixel on a modern quad-core CPU

Bad: specific to SSE, hard to tell what's going on at all!

```
void fast_blur(const Image &in, Image &blurred) {
 __m128i one_third = _mm_set1_epi16(21846);
 #pragma omp parallel for
 for (int yTile = 0; yTile < in.height(); yTile += 32) {
 __m128i a, b, c, sum, avg;
 __m128i tmp[(256/8)*(32+2)];
 for (int xTile = 0; xTile < in.width(); xTile += 256) {
 __m128i *tmpPtr = tmp;
 for (int y = -1; y < 32+1; y++) {
 const uint16_t *inPtr = &(in(xTile, yTile+y));
 for (int x = 0; x < 256; x += 8) {
 a = _mm_loadu_si128((__m128i*)(inPtr-1));
 b = _mm_loadu_si128((__m128i*)(inPtr+1));
 c = _mm_load_si128((__m128i*)(inPtr));
 sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
 avg = _mm_mulhi_epi16(sum, one_third);
 _mm_store_si128(tmpPtr++, avg);
 inPtr += 8;
 }
 }
 tmpPtr = tmp;
 for (int y = 0; y < 32; y++) {
 __m128i *outPtr = (__m128i *)(&(blurred(xTile, yTile+y)));
 for (int x = 0; x < 256; x += 8) {
 a = _mm_load_si128(tmpPtr+(2*256)/8);
 b = _mm_load_si128(tmpPtr+256/8);
 c = _mm_load_si128(tmpPtr++);
 sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
 avg = _mm_mulhi_epi16(sum, one_third);
 _mm_store_si128(outPtr++, avg);
 }
 }
 }
 }
}
```

Multi-core execution  
(partition image vertically)

Modified iteration order:  
256x32 block-major iteration  
(to maximize cache hit rate)

use of SIMD vector intrinsics

two passes fused into one:  
tmp data read from cache

Note: this implementation recomputes  
intermediate values. Why?

# Halide blur

## ■ Halide = two domain-specific co-languages

1. A purely functional DSL for defining image processing algorithms
2. A DSL for defining “schedules” for how to map these algorithms to a machine

```
Func halide_blur(Func in) {
 Func tmp, blurred;
 Var x, y, xi, yi;

 // The algorithm
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;

 return blurred;
}
```

Images are pure functions from integer coordinates (up to 4D domain) to values (color of corresponding pixels)

Algorithms are a series of functions (think: pipeline stages)

Functions (side-effect-free) map coordinates to values (in, tmp and blurred are functions)

**NOTE: execution order and storage are unspecified by abstraction!**  
**Implementation can evaluate, reevaluate, cache individual points as desired!**

# Halide blur

- Halide = two domain-specific co-languages
  1. A purely functional DSL for defining image processing algorithms
  2. A DSL for defining “schedules” for how to map these algorithms to a machine

```
Func halide_blur(Func in) {
 Func tmp, blurred;
 Var x, y, xi, yi;

 // The algorithm
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;

 // The schedule
 blurred.tile(x, y, xi, yi, 256, 32)
 .vectorize(xi, 8).parallel(y);
 tmp.chunk(x).vectorize(x, 8);

 return blurred;
}
```

When evaluating `blurred`, use 2D tiling order (loops named by `x, y, xi, yi`). Use tile size 256 x 32.

Vectorize the `xi` loop (8-wide), use threads to parallelize the `y` loop

Produce only chunks of `tmp` at a time. Vectorize the `x` (innermost) loop

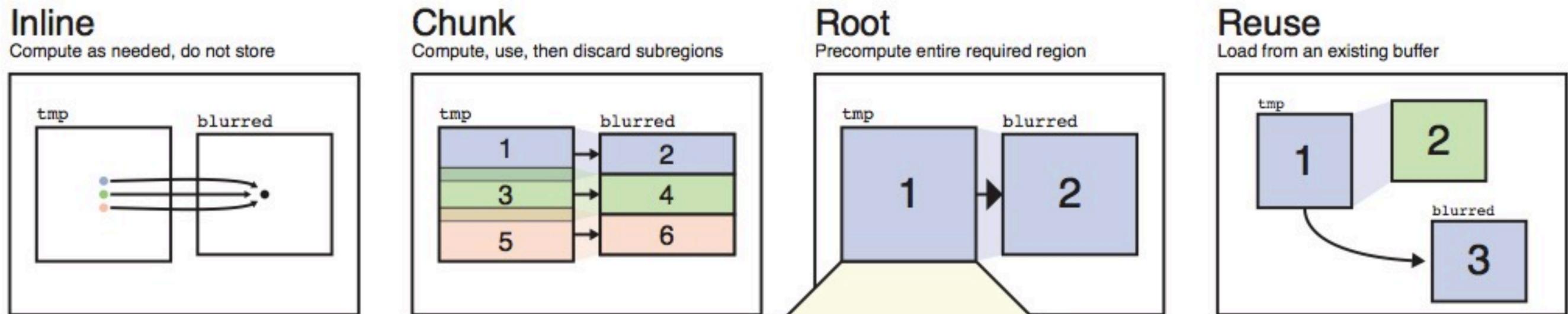
# Separation of algorithm from schedule

- **Key idea: separate specification of image processing algorithm (machine independent) from specification of schedule (machine dependent mapping)**
- **Given algorithm and schedule description, Halide can generate very high quality code for a target machine**
  - **Domain scope:**
    - **All computation over regular (up to 4D) grids**
    - **Only feed-forward pipelines (includes special support for reductions and fixed recursion depth)**
    - **All dependencies inferable by compiler**

# The Halide schedule

## ■ Halide schedule defines:

- Producer-consumer scheduling (four primitives shown below)



| Serial y, Serial x                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Serial x, Serial y | Serial y, Vectorized x | Parallel y, Vectorized x | Split x into $2x_0+x_i$ ,<br>Split y into $2y_0+y_i$ ,<br>Serial $y_0, x_0, y_i, x_i$ |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|------------------------|--------------------------|---------------------------------------------------------------------------------------|----|----|----|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|----|----|----|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| <table border="1"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr> <tr><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr> <tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td></tr> <tr><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td></tr> <tr><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td></tr> <tr><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td></tr> <tr><td>57</td><td>58</td><td>59</td><td>60</td><td>61</td><td>62</td><td>63</td><td>64</td></tr> </table> | 1                  | 2                      | 3                        | 4                                                                                     | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | <table border="1"> <tr><td>1</td><td>9</td><td>17</td><td>25</td><td>33</td><td>41</td><td>49</td><td>57</td></tr> <tr><td>2</td><td>10</td><td>18</td><td>26</td><td>34</td><td>42</td><td>50</td><td>58</td></tr> <tr><td>3</td><td>11</td><td>19</td><td>27</td><td>35</td><td>43</td><td>51</td><td>59</td></tr> <tr><td>4</td><td>12</td><td>20</td><td>28</td><td>36</td><td>44</td><td>52</td><td>60</td></tr> <tr><td>5</td><td>13</td><td>21</td><td>29</td><td>37</td><td>45</td><td>53</td><td>61</td></tr> <tr><td>6</td><td>14</td><td>22</td><td>30</td><td>38</td><td>46</td><td>54</td><td>62</td></tr> <tr><td>7</td><td>15</td><td>23</td><td>31</td><td>39</td><td>47</td><td>55</td><td>63</td></tr> <tr><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td></tr> </table> | 1 | 9 | 17 | 25 | 33 | 41 | 49 | 57 | 2 | 10 | 18 | 26 | 34 | 42 | 50 | 58 | 3 | 11 | 19 | 27 | 35 | 43 | 51 | 59 | 4 | 12 | 20 | 28 | 36 | 44 | 52 | 60 | 5 | 13 | 21 | 29 | 37 | 45 | 53 | 61 | 6 | 14 | 22 | 30 | 38 | 46 | 54 | 62 | 7 | 15 | 23 | 31 | 39 | 47 | 55 | 63 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | <table border="1"> <tr><td>1</td><td>2</td></tr> <tr><td>3</td><td>4</td></tr> <tr><td>5</td><td>6</td></tr> <tr><td>7</td><td>8</td></tr> <tr><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td></tr> <tr><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td></tr> </table> | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | <table border="1"> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td></tr> </table> | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | <table border="1"> <tr><td>1</td><td>2</td><td>5</td><td>6</td><td>9</td><td>10</td><td>13</td><td>14</td></tr> <tr><td>3</td><td>4</td><td>7</td><td>8</td><td>11</td><td>12</td><td>15</td><td>16</td></tr> <tr><td>17</td><td>18</td><td>21</td><td>22</td><td>25</td><td>26</td><td>29</td><td>30</td></tr> <tr><td>19</td><td>20</td><td>23</td><td>24</td><td>27</td><td>28</td><td>31</td><td>32</td></tr> <tr><td>33</td><td>34</td><td>37</td><td>38</td><td>41</td><td>42</td><td>45</td><td>46</td></tr> <tr><td>35</td><td>36</td><td>39</td><td>40</td><td>43</td><td>44</td><td>47</td><td>48</td></tr> <tr><td>49</td><td>50</td><td>53</td><td>54</td><td>57</td><td>58</td><td>61</td><td>62</td></tr> <tr><td>51</td><td>52</td><td>55</td><td>56</td><td>59</td><td>60</td><td>63</td><td>64</td></tr> </table> | 1 | 2 | 5 | 6 | 9 | 10 | 13 | 14 | 3 | 4 | 7 | 8 | 11 | 12 | 15 | 16 | 17 | 18 | 21 | 22 | 25 | 26 | 29 | 30 | 19 | 20 | 23 | 24 | 27 | 28 | 31 | 32 | 33 | 34 | 37 | 38 | 41 | 42 | 45 | 46 | 35 | 36 | 39 | 40 | 43 | 44 | 47 | 48 | 49 | 50 | 53 | 54 | 57 | 58 | 61 | 62 | 51 | 52 | 55 | 56 | 59 | 60 | 63 | 64 |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  | 3                      | 4                        | 5                                                                                     | 6  | 7  | 8  |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 10                 | 11                     | 12                       | 13                                                                                    | 14 | 15 | 16 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 17                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 18                 | 19                     | 20                       | 21                                                                                    | 22 | 23 | 24 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 25                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 26                 | 27                     | 28                       | 29                                                                                    | 30 | 31 | 32 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 33                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 34                 | 35                     | 36                       | 37                                                                                    | 38 | 39 | 40 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 41                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 42                 | 43                     | 44                       | 45                                                                                    | 46 | 47 | 48 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 49                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 50                 | 51                     | 52                       | 53                                                                                    | 54 | 55 | 56 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 57                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 58                 | 59                     | 60                       | 61                                                                                    | 62 | 63 | 64 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 9                  | 17                     | 25                       | 33                                                                                    | 41 | 49 | 57 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 10                 | 18                     | 26                       | 34                                                                                    | 42 | 50 | 58 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 11                 | 19                     | 27                       | 35                                                                                    | 43 | 51 | 59 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 12                 | 20                     | 28                       | 36                                                                                    | 44 | 52 | 60 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 13                 | 21                     | 29                       | 37                                                                                    | 45 | 53 | 61 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 14                 | 22                     | 30                       | 38                                                                                    | 46 | 54 | 62 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 15                 | 23                     | 31                       | 39                                                                                    | 47 | 55 | 63 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 16                 | 24                     | 32                       | 40                                                                                    | 48 | 56 | 64 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 4                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 6                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 8                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 10                 |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 11                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 12                 |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 13                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 14                 |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 15                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 16                 |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  |                        |                          |                                                                                       |    |    |    |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                  | 5                      | 6                        | 9                                                                                     | 10 | 13 | 14 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 4                  | 7                      | 8                        | 11                                                                                    | 12 | 15 | 16 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 17                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 18                 | 21                     | 22                       | 25                                                                                    | 26 | 29 | 30 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 19                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 20                 | 23                     | 24                       | 27                                                                                    | 28 | 31 | 32 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 33                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 34                 | 37                     | 38                       | 41                                                                                    | 42 | 45 | 46 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 35                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 36                 | 39                     | 40                       | 43                                                                                    | 44 | 47 | 48 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 49                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 50                 | 53                     | 54                       | 57                                                                                    | 58 | 61 | 62 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 51                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 52                 | 55                     | 56                       | 59                                                                                    | 60 | 63 | 64 |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                                    |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |                                                                                                                                                                                                                                                                             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |   |   |   |   |    |    |    |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

# The Halide schedule

## ■ Halide schedule defines:

1. **Producer-consumer scheduling (four primitives shown below)**
2. **Computation order and parallelization within a stage**
  - **Order of point-wise execution: e.g., `serial(y)`, `serial(x)`**
  - **How to parallelize: e.g., `parallel(y)`, `vectorize(x)`**

# Halide results

## ■ Camera RAW processing pipeline

(Convert RAW sensor data to RGB image)

- **Original: 463 lines of hand-tuned ARM assembly**
- **Halide: 2.75x less code, 5% faster**



- Denoise
- Demosaic
- Color Correct
- Tone Curve



## ■ Bilateral filter

(Common image filtering operation used in many applications)

- **Original 122 lines of C++**
- **Halide: 34 lines algorithm + 6 lines schedule**
  - **CPU implementation: 5.9x faster**
  - **GPU implementation: 2x faster than hand-written CUDA**



- Grid Construction
- Blurring
- Slicing



**Takeaway: Halide is not magic, but its abstractions allow rapid exploration of optimization space, allowing programmer to reach optimal points quickly**

# Many other recent domain-specific programming systems



Less domain specific than examples given today,  
but still designed specifically for:  
data-parallel computations on big data for  
distributed systems (“Map-Reduce”)



DSL for graph-based machine learning operations

Also see **Green-Marl**

(another DSL for describing operations on graphs)



Model-view-controller paradigm for  
web-applications

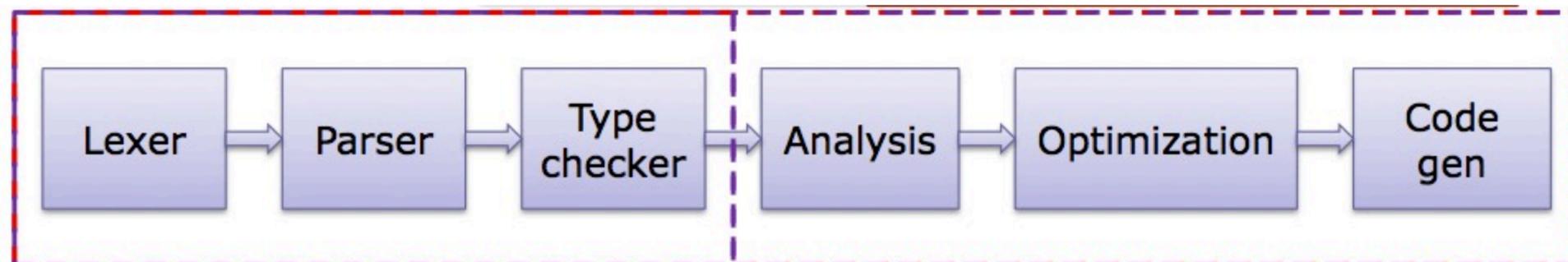
**Ongoing efforts in many domains...**

# Domain-specific programming system development

- **Can develop DSL as a stand-alone language**
  - Graphics shading languages
  - MATLAB, SQL
- **“Embed” DSL in an existing generic language**
  - e.g., C++ library (GraphLab, OpenGL host-side API, Map-Reduce)
  - Lizst syntax above was all valid Scala
- **Active research idea:**
  - Design generic languages that have facilities that assist rapid embedding of new domain-specific languages

# Facilitating development of new domain-specific languages

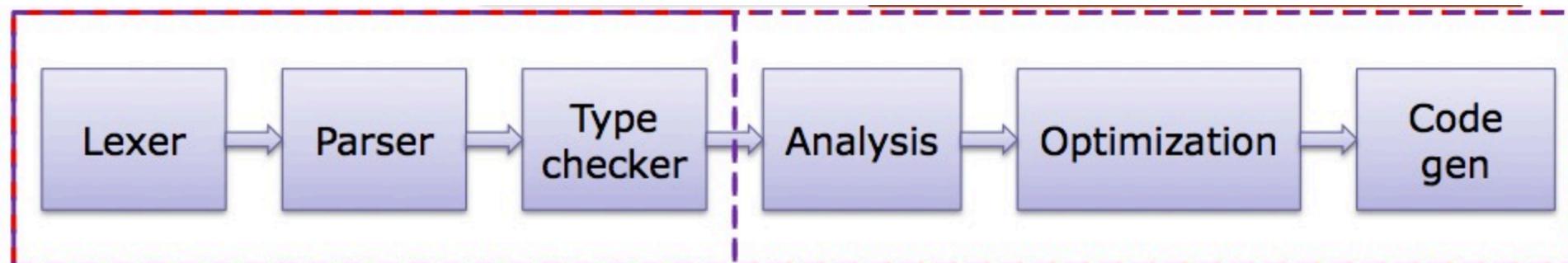
“Embed” domain-specific language in generic, flexible embedding language



## Typical Compiler

Stand-alone domain-special language must implement everything

## “Modular staging” approach:



Domain language adopts front-end from highly expressive embedding language

Leverage techniques like operator overloading, modern OOP (traits), type inference, closures, to make embedding language syntax appear native:

Liszt code shown before was actually valid Scala!

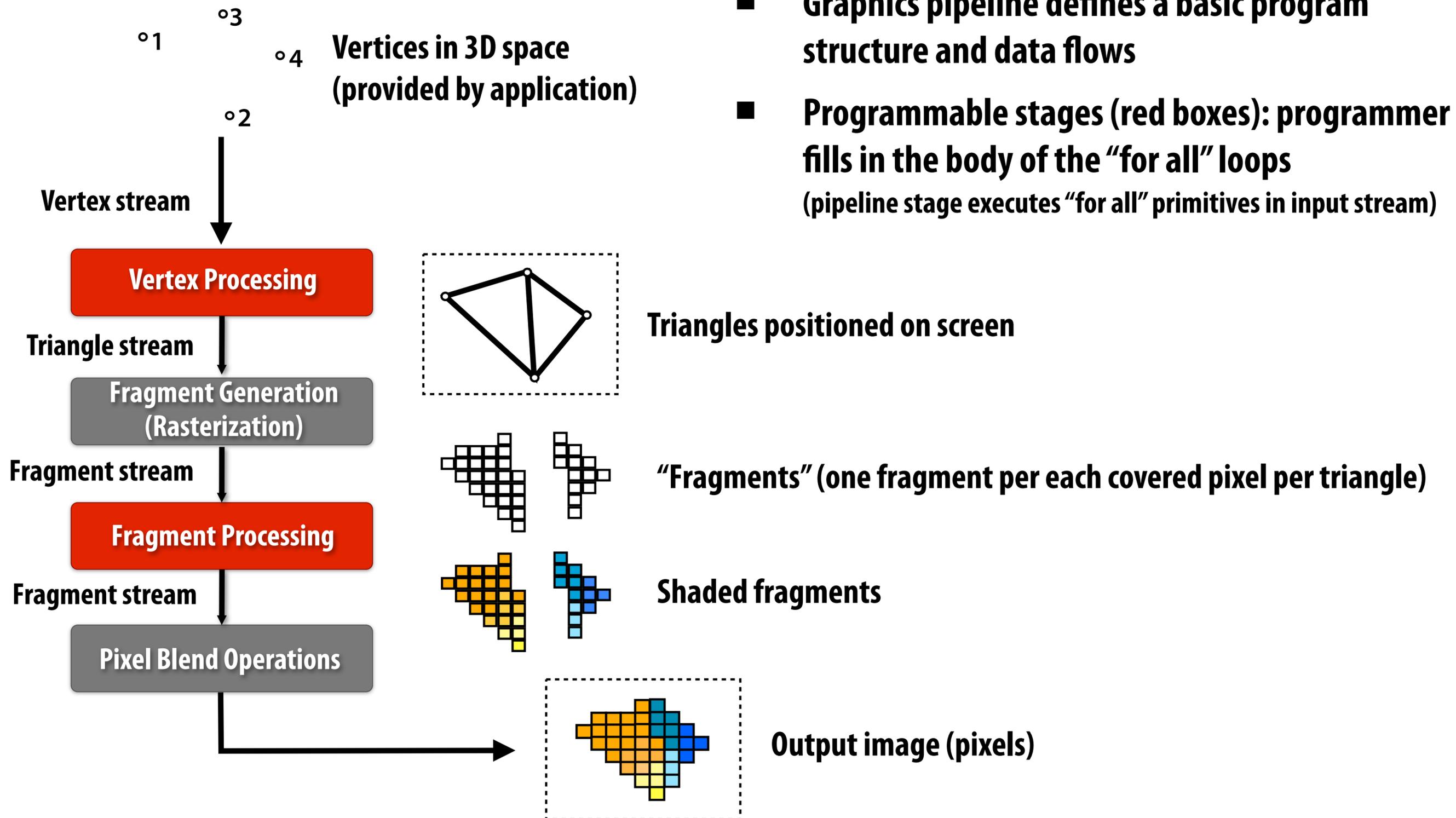
But customizes intermediate representation (IR) and participates in backend optimization and code-generation phases (exploiting domain knowledge while doing so)

# Summary

- **Modern machines: parallel, heterogeneous**
  - Only way to increase compute capability in power-constrained world
- **Most software uses very little of peak capability of machine**
  - Very challenging to tune programs to these machines
  - Tuning efforts are not portable across machines
- **Domain-specific programming environments trade-off generality to achieve productivity, performance, and portability**
  - Case studies today: Liszt, Halide, OpenGL (see bonus slides)
  - Common trait: languages provide abstractions that make dependencies known
    - Understanding dependencies is necessary but not sufficient: need domain restrictions and domain knowledge for system to synthesize efficient implementations

**Bonus slides!**  
**DSL Example 3:**  
**OpenGL: a domain-specific system for 3D rendering**

# OpenGL graphics pipeline



- Key abstraction: graphics pipeline
- Graphics pipeline defines a basic program structure and data flows
- Programmable stages (red boxes): programmer fills in the body of the “for all” loops (pipeline stage executes “for all” primitives in input stream)

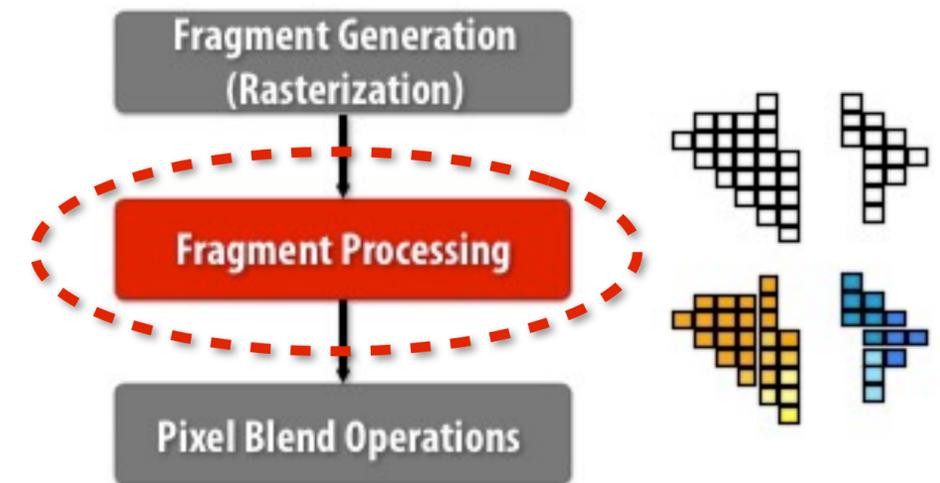
# Fragment “shader” program

**HLSL shader program: defines behavior of fragment processing stage**  
**Executes once per pixel covered by each triangle**

**Input: a “fragment”: information about the triangle at the pixel**  
**Output: RGBA color (float4 datatype)**

```
sampler mySamp;
Texture2D<float3> myTex;
float3 lightDir;

float4 diffuseShader(float3 norm, float2 uv)
{
 float3 kd;
 kd = myTex.sample(mySamp, uv);
 kd *= clamp(dot(lightDir, norm), 0.0, 1.0);
 return float4(kd, 1.0);
}
```



## Productivity:

- **SPMD program: no explicit parallelism**
- **Implicit parallelism: programmer writes no loops over fragments (think of shader as a loop body)**
- **Code runs independently for each input fragment (no loops = impossible to express a loop dependency)**

## Performance:

- **SPMD program compiles to wide SIMD processing on GPU**
- **Work for many fragments dynamically balanced onto GPU cores**
- **Performance Portability:**
  - **Scales to GPUs with different # of cores**
  - **SPMD abstraction compiles to different SIMD widths (NVIDIA=32, AMD=64,**

# Special language primitive for texture mapping

```
sampler mySamp;
Texture2D<float3> myTex;
float3 lightDir;

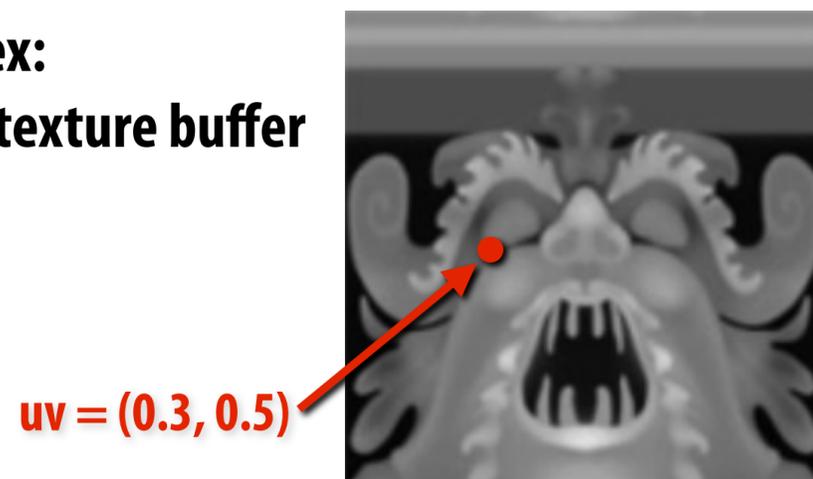
float4 diffuseShader(float3 norm, float2 uv)
{
 float3 kd;
 kd = myTex.sample(mySamp, uv);
 kd *= clamp(dot(lightDir, norm), 0.0, 1.0);
 return float4(kd, 1.0);
}
```

Intuitive abstraction: represents a texture lookup like an array access with a 2D floating point index.

Texture fetch semantics: sample from `myTex` at coordinate `uv` and filter using scheme (e.g., bilinear filtering) defined by `mySamp`.

Result of mapping texture onto plane, viewed with perspective

myTex:  
NxN texture buffer

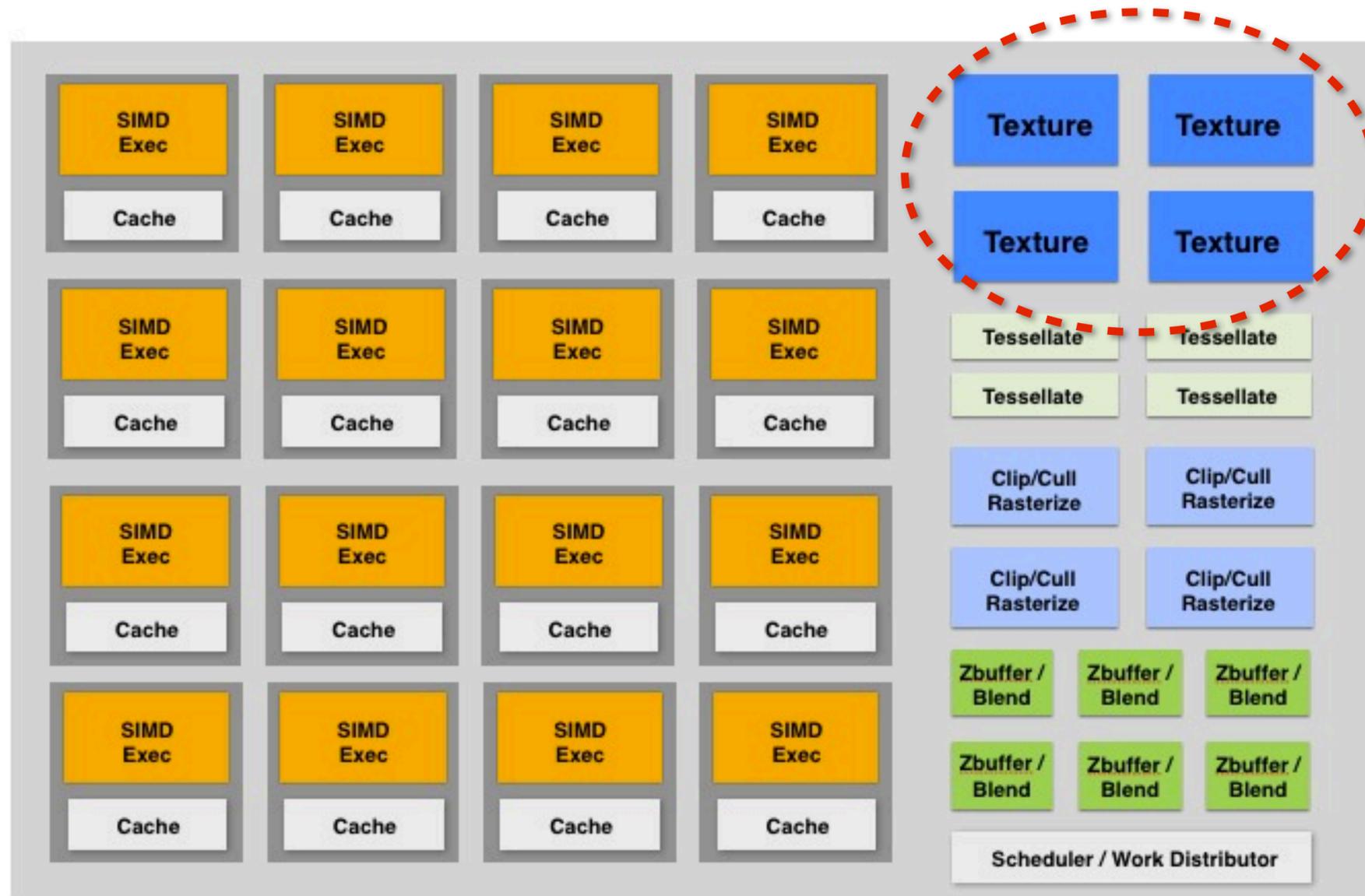


# Texture mapping is expensive (and performance critical)

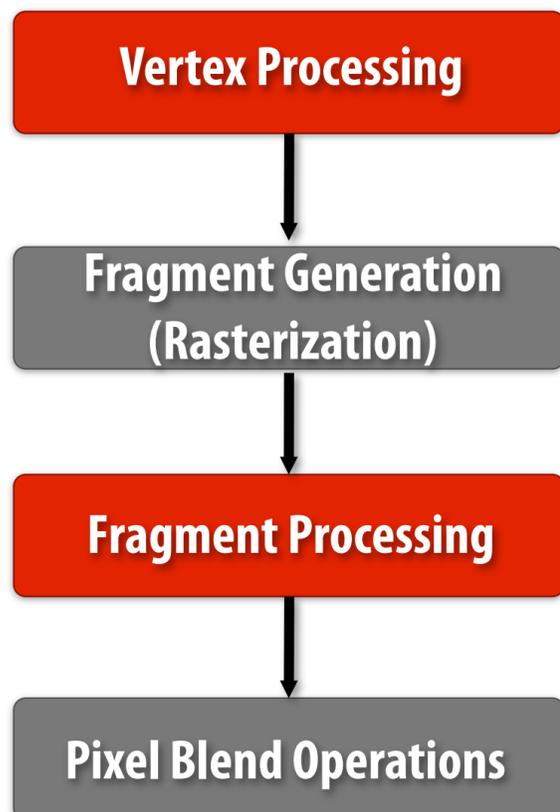
- **Texture mapping is a filtering operation (more than an array lookup: see 15-462)**
  - **If implemented in software: ~ 50 instructions, multiple conditionals**
  - **Read at least 8 values from texture map, blend them together**
    - **Unpredictable data access, little temporal locality**
- **Typical shader program performs multiple texture lookups**
- **Texture mapping is one of the most computationally demanding AND bandwidth intensive aspects of the graphics pipeline**
  - **Resources for texturing must run near 100% efficiency**
  - **Not surprising it is encapsulated in its own primitive**

# Performance: texture mapping

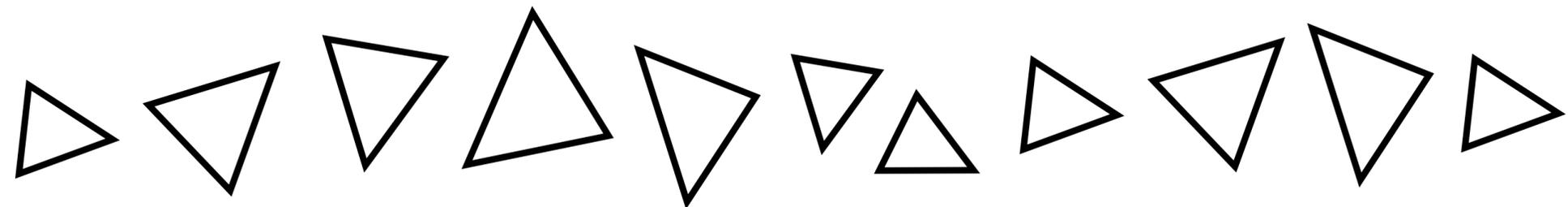
- Highly multi-threaded cores hide latency of memory access  
(texture primitive = source of long memory stalls is explicit in programming model)
- Fixed-function HW to perform texture mapping math
- Special-cache designs to capture reuse, exploit read-only access to texture data



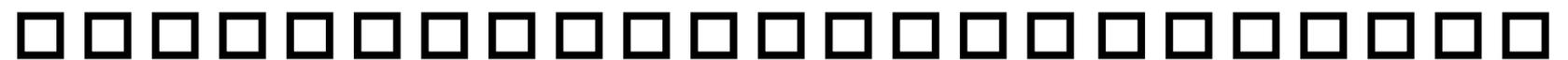
# Performance: global application orchestration



## Parallel work:



Hundreds of thousands of triangles



Millions of fragments to shade



Millions of shaded fragments to blend into output image

Efficiently scheduling all this parallel work onto the GPU's heterogeneous pool of resources (while also respecting the ordering requirements of the OpenGL programming model) is challenging.

Each GPU vendor uses its own custom strategy (high-level abstraction allows for different implementations)

# OpenGL summary

## ■ Productivity:

- High-level, intuitive abstractions (taught to undergrads in intro graphics class)
- Application implements kernels for triangles, vertices, and fragments
- Specific primitives for key functions like texture mapping

## ■ Portability

- Runs across wide range of GPUs: low-end integrated, high-end discrete, mobile
- Has allowed significant hardware innovation without impacting programmer

## ■ High-Performance

- Abstractions designed to map efficiently to hardware  
(proposed new features disallowed if they do not!)
- Encapsulating expensive operations as unique pipeline stages or built-in functions  
facilitates fixed-function implementations (texture, rasterization, frame-buffer blend)
- Utilize domain-knowledge in optimizing performance / mapping to hardware
  - Skip unnecessary work, e.g., if a triangle it is determined to be behind another, don't generate and shade its fragments
  - Non-overlapping fragments are independent despite ordering constraint
  - Interstage queues/buffers are sized based on expected triangle sizes
  - Use pipeline structure to make good scheduling decisions, set work priorities