Lecture 26:
Parallel Deep Network Training
“Eat your veggies and study for Exam 2.”
- Corynne Elliot, on ending the semester well.
Announcements

- Exam 2 Review Session, Saturday at 4pm in Rashid
  - Please come with questions

- I have an additional office hours on Sunday at 4pm

- Exam 2: Monday, at 6:15 in Rashid
  - Closed notes (one post-it note rule applies)
  - Covers all material from memory consistency onward

Because there were a few issues with understanding consistency on exam 1
Kayvon’s three incredibly useful exam 2 hints

1
2
3
Professor classification network

Classifies professors as easy, mean, boring, or nerdy based on their appearance.

Input: image of a professor

Output: probability of label

Recall from last time: 10’s-100’s of millions of parameters
Our model:
- Max-pooling layers follow first, second, and fifth convolutional layers.
- The number of neurons in each layer is given by: 253440, 186624, 64896, 64896, 43264, 4096, 4096, 1000.

Easy: 0.26
Mean: 0.08
Boring: 0.14
Nerdy: 0.52
Where did the parameters come from?
Training data (ground truth answers)
Professor classification network

New image of Kayvon (not in training set)

Easy: 0.0
Mean: 0.0
Boring: 0.0
Nerdy: 1.0

Ground truth (what the answer should be)

Easy: 0.26
Mean: 0.08
Boring: 0.14
Nerdy: 0.52

Network output
## Error (loss)

### Ground truth:
(what the answer should be)

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy</td>
<td>0.0</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0</td>
</tr>
<tr>
<td>Boring</td>
<td>0.0</td>
</tr>
<tr>
<td>Nerdy</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Network output: *

<table>
<thead>
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<td>Nerdy</td>
<td>0.52</td>
</tr>
</tbody>
</table>

### Common example: softmax loss:

\[
L = -\log \left( \frac{e^{f_c}}{\sum_j e^{f_j}} \right)
\]

* In practice a network using a softmax classifier outputs unnormalized, log probabilities \((f_j)\), but I’m showing a probability distribution above for clarity.
Training

Goal of training: learning good values of network parameters so that network outputs the correct classification result for any input image

Idea: minimize loss for all the training examples (for which the correct answer is known)

\[ L = \sum_i L_i \quad \text{(total loss for entire training set is sum of losses } L_i \text{ for each training example } x_i) \]

Intuition: if the network gets the answer correct for a wide range of training examples, then hopefully it has learned parameter values that yield the correct answer for future images as well.
Intuition: gradient descent

Say you had a function $f$ that contained a hidden parameters $p_1$ and $p_2$: $f(x_i)$

And for some input $x_i$, your training data says the function should output 0.

But for the current values of $p_1$ and $p_2$, it currently outputs 10.

$$f(x_i, p_1, p_2) = 10$$

And say I also gave you expressions for the derivative of $f$ with respect to $p_1$ and $p_2$ so you could compute their value at $x_i$.

$$\frac{df}{dp_1} = 2 \quad \frac{df}{dp_2} = -5$$

$$\nabla f = [2, -5]$$

How might you adjust the values $p_1$ and $p_2$ to reduce the error for this training example?
Basic gradient descent

while (loss too high):
  for each item $x_i$ in training set:
    grad += evaluate_loss_gradient($f$, loss_func, params, $x_i$)
  params += -grad * step_size;

Mini-batch stochastic gradient descent (mini-batch SGD): choose a random (small) subset of the training examples to compute gradient in each iteration of the while loop

How to compute $df/dp$ for a complex neural network with millions of parameters?
Derivatives using the chain rule

\[ f(x, y, z) = (x + y)z = az \]

Where: \( a = x + y \)

\[ \frac{df}{da} = z \quad \frac{da}{dx} = 1 \quad \frac{da}{dy} = 1 \]

So, by the derivative chain rule:

\[ \frac{df}{dx} = \frac{df}{da} \frac{da}{dx} = z \]

Red = output of node
Blue = \( \frac{df}{d\text{node}} \)
Backpropagation

Red = output of node
Blue = df/dnode

Recall:
\[
\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx}
\]

\[
g(x, y) = x + y
\]
\[
\frac{dg}{dx} = 1, \quad \frac{dg}{dy} = 1
\]

\[
g(x, y) = \max(x, y)
\]
\[
\frac{dg}{dx} = 1, \text{ if } x > y
\]
\[
\frac{dg}{dx} = 0, \text{ otherwise}
\]

\[
g(x, y) = xy
\]
\[
\frac{dg}{dx} = y, \quad \frac{dg}{dy} = x
\]
Backpropagating through single unit

\[ f(x_0, x_1, x_2, x_3) = \max \left( 0, \sum_i x_i w_i + b \right) \]

Recall: behavior of unit:

let \( y = \begin{cases} 
  10, & \text{if upper input to max is > 0} \\
  0, & \text{otherwise} 
\end{cases} \)

Observe: output of prior layer (\( x_i \)'s) and output of this unit must be retained in order to compute weight gradients for this unit during backprop.
Multiple uses of an input variable

Gradients from each use of variable sum:

Here:

\[
\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx}
\]

\[
= 10 \frac{dg}{dx}
\]

\[
= 10(2x + 1)
\]

\[
= 10(10 + 1) = 110
\]

\[
g(x, y) = (x + y) + x \times x = a + b
\]

\[
a = 1, \quad b = 2x
\]

\[
\frac{dg}{dx} = \frac{dg}{da} \frac{da}{dx} + \frac{dg}{db} \frac{db}{dx} = 2x + 1
\]

Implication: backpropagation through all units in a convolutional layer adds gradients computed from each unit to the overall gradient for the shared weights
Backpropagation: matrix form

\[ y = Xw \]

\[ \frac{dL}{dw} = \frac{dL}{dy} \frac{dy}{dw} \]

\[ \frac{dy_j}{dw_i} = X_{ji} \]

\[ \frac{dL}{dw_i} = \sum_j \frac{dL}{dy_j} \frac{dy_j}{dw_i} \]

\[ = \sum_j \frac{dL}{dy_j} X_{ji} \]

Therefore:

\[ \frac{dL}{dw} = X^T \frac{dL}{dy} \]
Backpropagation through the entire professor classification network

For each training example \( x_i \) in mini-batch:

- Perform forward evaluation to compute loss for \( x_i \)
  
  Note: must retain all layer outputs + output gradients (needed to compute weight gradients during backpropagation)

- Compute gradient of loss w.r.t. final layer’s outputs

- Backpropagate gradient to compute gradient of loss w.r.t. all network parameters

- Accumulate gradients (over all images in batch)

- Update all parameter values: \( w_{i\_new} = w_{i\_old} - \text{step\_size} \times \text{grad}_i \)
# VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

<table>
<thead>
<tr>
<th>Layer Type</th>
<th>Input Size</th>
<th>Output Size</th>
<th>Weights Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: 224 x 224 RGB image</td>
<td>—</td>
<td>224x224x3</td>
<td>—</td>
</tr>
<tr>
<td>Conv: (3x3x3) x 64</td>
<td>6.5 KB</td>
<td>224x224x64</td>
<td>150K</td>
</tr>
<tr>
<td>Conv: (3x3x64) x 64</td>
<td>144 KB</td>
<td>224x224x64</td>
<td>12.3 MB</td>
</tr>
<tr>
<td>Maxpool</td>
<td>—</td>
<td>112x112x64</td>
<td>3.1 MB</td>
</tr>
<tr>
<td>Conv: (3x3x64) x 128</td>
<td>228 KB</td>
<td>112x112x128</td>
<td>6.2 MB</td>
</tr>
<tr>
<td>Conv: (3x3x128) x 128</td>
<td>576 KB</td>
<td>112x112x128</td>
<td>6.2 MB</td>
</tr>
<tr>
<td>Maxpool</td>
<td>—</td>
<td>56x56x128</td>
<td>1.5 MB</td>
</tr>
<tr>
<td>Conv: (3x3x128) x 256</td>
<td>1.1 MB</td>
<td>56x56x256</td>
<td>3.1 MB</td>
</tr>
<tr>
<td>Conv: (3x3x256) x 256</td>
<td>2.3 MB</td>
<td>56x56x256</td>
<td>3.1 MB</td>
</tr>
<tr>
<td>Conv: (3x3x256) x 256</td>
<td>2.3 MB</td>
<td>56x56x256</td>
<td>3.1 MB</td>
</tr>
<tr>
<td>Maxpool</td>
<td>—</td>
<td>28x28x256</td>
<td>766 KB</td>
</tr>
<tr>
<td>Conv: (3x3x256) x 512</td>
<td>4.5 MB</td>
<td>28x28x512</td>
<td>1.5 MB</td>
</tr>
<tr>
<td>Conv: (3x3x512) x 512</td>
<td>9 MB</td>
<td>28x28x512</td>
<td>1.5 MB</td>
</tr>
<tr>
<td>Conv: (3x3x512) x 512</td>
<td>9 MB</td>
<td>28x28x512</td>
<td>1.5 MB</td>
</tr>
<tr>
<td>Maxpool</td>
<td>—</td>
<td>14x14x512</td>
<td>383 KB</td>
</tr>
<tr>
<td>Conv: (3x3x512) x 512</td>
<td>9 MB</td>
<td>14x14x512</td>
<td>383 KB</td>
</tr>
<tr>
<td>Conv: (3x3x512) x 512</td>
<td>9 MB</td>
<td>14x14x512</td>
<td>383 KB</td>
</tr>
<tr>
<td>Conv: (3x3x512) x 512</td>
<td>9 MB</td>
<td>14x14x512</td>
<td>383 KB</td>
</tr>
<tr>
<td>Maxpool</td>
<td>—</td>
<td>7x7x512</td>
<td>98 KB</td>
</tr>
<tr>
<td>Fully-connected 4096</td>
<td>392 MB</td>
<td>4096</td>
<td>16 KB</td>
</tr>
<tr>
<td>Fully-connected 4096</td>
<td>64 MB</td>
<td>4096</td>
<td>16 KB</td>
</tr>
<tr>
<td>Fully-connected 1000</td>
<td>15.6 MB</td>
<td>1000</td>
<td>4 KB</td>
</tr>
</tbody>
</table>

**Unlike forward evaluation:**
1. must store outputs and gradient of outputs
2. cannot immediately free outputs once consumed by next level of network

**inputs/outputs get multiplied by mini-batch size**

Many implementations also store gradient “momentum” as well (multiply by 3)

**Must also store per-weight gradients**
SGD workload

while (loss too high):
  for each item x_i in mini-batch:
    grad += evaluate_loss_gradient(f, loss_func, params, x_i)
  params += -grad * step_size;

At first glance, this loop is sequential (each step of “walking downhill” depends on previous)

Parallel across images

large computation with its own parallelism (but working set may not fit on single machine)

trivial data-parallel over parameters
Deep network training workload

- **Huge computational expense**
  - Must evaluate the network (forward and backward) for millions of training images
  - Must iterate for many iterations of gradient descent (100’s of thousands)
  - Training modern networks takes days

- **Large memory footprint**
  - Must maintain network layer outputs from forward pass
  - Additional memory to store gradients for each parameter
  - Recall parameters for popular VGG-16 network require ~500 MB of memory (training requires GBs of memory for academic networks)
  - Scaling to larger networks requires partitioning network across nodes to keep network intermediates in memory

- **Dependencies /synchronization (not embarrassingly parallel)**
  - Each parameter update step depends on previous
  - Many units contribute to same parameter gradients (fine-scale reduction)
  - Different images in mini batch contribute to same parameter gradients
Data-parallel training (across images)

for each item $x_i$ in mini-batch:
  grad += evaluate_loss_gradient(f, loss_func, params, $x_i$)
params += -grad * step_size;

Consider parallelization of the outer for loop across machines in a cluster

partition mini-batch across nodes
for each item $x_i$ in mini-batch assigned to local node:
  // just like single node training
  grad += evaluate_loss_gradient(f, loss_func, params, $x_i$)
barrier();
sum reduce gradients, communicate results to all nodes
barrier();
update copy of parameter values
Challenges of computing at cluster scale

- Slow communication between nodes
  - Clusters do not feature high-performance interconnects typical of supercomputers

- Nodes with different performance (even if machines are the same)
  - Workload imbalance at barriers (sync points between nodes)

Modern solution: exploit characteristics of SGD using asynchronous execution!
Parameter server design

Pool of worker nodes

Worker Node 0  Worker Node 1  Worker Node 2  Worker Node 3

Parameter Server

Parameter Server [Li OSDI14]
Google’s DistBelief [Dean NIPS12]
Microsoft’s Project Adam [Chilimbi OSDI14]
Training data partitioned among workers

Pool of worker nodes

- Training data
  - Worker Node 0
  - Worker Node 1
  - Worker Node 2
  - Worker Node 3

Parameter values (v0)

Parameter Server
Copy of parameters sent to workers

Pool of worker nodes

Parameter server

Worker Node 0
- Training data
- Local copy of parameters (v0)

Worker Node 1
- Training data
- Local copy of parameters (v0)

Worker Node 2
- Training data
- Local copy of parameters (v0)

Worker Node 3
- Training data
- Local copy of parameters (v0)

Parameter values (v0)
Workers independently compute local “subgradients"
Worker sends subgradient to parameter server

Pool of worker nodes

Worker Node 0
- training data
- local copy of parameters (v0)
- local subgradients

Worker Node 1
- training data
- local copy of parameters (v0)
- local subgradients

Worker Node 2
- training data
- local copy of parameters (v0)
- local subgradients

Worker Node 3
- training data
- local copy of parameters (v0)
- local subgradients

Parameter Server
- parameter values (v0)

Red arrow from Worker Node 1 to Parameter Server labeled as subgradient.
Server updates global parameter values based on subgradient

params += -subgrad * step_size;
Updated parameters sent to worker
Worker proceeds with another gradient computation step

Note:
Node 1 is operating on different set of parameter values than other nodes
Those parameter values were computed without gradient information from the other nodes
Updated parameters sent to worker (again)
Worker continues with updated parameters

Worker Node 0
- training data
- local copy of parameters (v0)
- local subgradients

Worker Node 1
- training data
- local copy of parameters (v1)
- local subgradients

Worker Node 2
- training data
- local copy of parameters (v0)
- local subgradients

Worker Node 3
- training data
- local copy of parameters (v2)
- local subgradients

Parameter Server
- parameter values (v2)

params v2
Summary: asynchronous parameter update

- Idea: avoid global synchronization on all parameter updates between each SGD iteration
  - Design reflects realities of cluster computing:
    - Slow interconnects
    - Unpredictable machine performance

- Solution: asynchronous (and partial) subgradient updates

- Will impact convergence of SGD
  - Node N working on iteration i may not have parameter values that result the results of the i-1 prior SGD iterations
Bottleneck?

What if there is heavy contention for parameter server?

Worker Node 0

training data
local copy of parameters (v0)
local subgradients

Worker Node 1

training data
local copy of parameters (v1)
local subgradients

Worker Node 2

training data
local copy of parameters (v0)
local subgradients

Worker Node 3

training data
local copy of parameters (v2)
local subgradients

Parameter Server

parameter values (v2)
Shard the parameter server

Partition parameters across servers
Worker sends chunk of subgradients to owning parameter server

- **Worker Node 0**: training data, local copy of parameters (v0), local subgradients
- **Worker Node 1**: training data, local copy of parameters (v1), local subgradients
- **Worker Node 2**: training data, local copy of parameters (v0), local subgradients
- **Worker Node 3**: training data, local copy of parameters (v2), local subgradients

- **Parameter Server 0**: parameter values (chunk 0)
- **Parameter Server 1**: parameter values (chunk 1)

Reduces data transmission load on individual servers (less important: also reduces cost of parameter update)
What if model parameters do not fit on one worker?

Recall high footprint of training large networks (particularly with large mini-batch sizes)
Model parallelism

Partition network parameters across nodes (spatial partitioning to reduce communication)

Reduce internode communication through network design:
- Use small spatial convolutions (1x1 convolutions)
- Reduce/shrink fully-connected layers

Convolutional layers: only need to communicate outputs near spatial partition

Fully-connected layers: all data owned by a node must be communicated to other nodes
Training data-parallel and model-parallel execution

Worker Node 0
- training data
- local copy of parameters (v1): chunk 0
- local subgradients chunk 0

Worker Node 1
- training data
- local copy of parameters (v1): chunk 1
- local subgradients chunk 1

Worker Node 2
- training data
- local copy of parameters (v0): chunk 0
- local subgradients chunk 0

Worker Node 3
- training data
- local copy of parameters (v0): chunk 1
- local subgradients chunk 1

Parameter Server 0
- parameter values (chunk 0)

Parameter Server 1
- parameter values (chunk 1)

Working on subgradient computation for a single copy of the model

Find-grained communication of layer outputs, subgradients, etc.
Using supercomputers for training?

- Fast interconnects critical for model-parallel training
  - Fine-grained communication of outputs and gradients

- Fast interconnect diminishes need for async training algorithms
  - Avoid randomness in training due to computation schedule (there remains randomness due to SGD algorithm)

OakRidge Titan Supercomputer

NVIDIA DGX-1: 8 Pascal GPUs connected via high speed NV-Link interconnect
Parallelizing mini-batch on one machine

for each item $x_i$ in mini-batch:
  grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;

Consider parallelization of the outer for loop across cores

Good: completely independent computations (until gradient reduction)
Bad: complete duplication of parameter gradient state (100’s MB per core)
Asynchronous update on one node

for each item $x_i$ in mini-batch:
  grad += evaluate_loss_gradient($f$, $loss_func$, params, $x_i$)
params += -grad * step_size;

Cores update shared set of gradients.
Skip taking locks / synchronizing across cores: perform “approximate reduction”

Project Adam [Chilimbi OSDI14]
Summary: training large networks in parallel

- Most systems rely on asynchronous update to efficiently used clusters of commodity machines
  - Modification of SGD algorithm to meet constraints of modern parallel systems
  - Open question: effects on convergence are problem dependent and not particularly well understood
  - Tighter integration / faster interconnects may provide alternative to these methods (facilitate tightly orchestrated solutions much like supercomputing applications)

- Open question: how big of networks are needed?
  - Last time we saw that >90% of connections could be removed without significant impact on quality of network
  - High-performance training of deep networks is an interesting example of constant iteration of algorithm design and parallelization strategy (a key theme of this course! recall the original grid solver example!)
Go study!