Problem 1: Optimizing a Multi-Threaded Program (10 pts)

Your friend writes the following multi-threaded C++ program that combines two images `image1` and `image2`. The implementation uses three threads, and each thread is responsible for processing a single channel (red, green, or blue) of the image. Notice that this processing requires the thread to loop over the image data `MAX_ITERS` times.

```cpp
struct Pixel {
    float r, g, b;
};

#define MAX_ITERS 1000
#define IMAGE_SIZE 64 * 64
float my_func(float, float);
Pixel *image1, *image2;

void workerR() {
    for (int iters=0; iters<MAX_ITERS; iters++)
        for (int i=0; i<IMAGE_SIZE; i++)
            result[i].r += my_func(image1[i].r, image2[i].r);
}

void workerG() {
    for (int iters=0; iters<MAX_ITERS; iters++)
        for (int i=0; i<IMAGE_SIZE; i++)
            result[i].g += my_func(image1[i].g, image2[i].g);
}

void workerB() {
    for (int iters=0; iters<MAX_ITERS; iters++)
        for (int i=0; i<IMAGE_SIZE; i++)
            result[i].b += my_func(image1[i].b, image2[i].b);
}

int main() {
    image1 = new Pixel[IMAGE_SIZE];
    image2 = new Pixel[IMAGE_SIZE];
    result = new Pixel[IMAGE_SIZE];

    // ... initialize result, image1, image2 here ...

    pthread_t t0, t1;
    pthread_create(&t0, NULL, workerR, NULL);
    pthread_create(&t1, NULL, workerG, NULL);
    workerB();
    pthread_join(t0, NULL);
    pthread_join(t1, NULL);

    // ... use 'result' image here ...
}
```
A. (5 pts) Your friend runs this program on the cache coherent quad-core Intel processor. Given that the problem is embarrassingly parallel and assuming the images are small enough that all three images can fit in the private L2 cache of each core, your friend expects near perfect (3×) speedup. They are shocked when they don’t obtain a good speedup. What is the cause of this suboptimal behavior?

B. (5 pts) Modify the program to correct the performance problem you identified in part A. You are allowed to modify the data structures used in the code but you are not allowed to change what computations are performed by each thread. That is, workerR must still process the red channel of the image, workerG must still process the green channel, etc. You only need to describe your solution in text or pseudocode (compilable C++ is not required). (Hint: there is a very simple change.)
Problem 2: Particle Simulation (10 pts)

Consider the following code that uses a simple $O(N^2)$ algorithm to compute forces due to gravitational interactions between all $N$ particles in a particle simulation. One important detail of this algorithm is that force computation is symmetric ($\text{gravity}(i,j) = \text{gravity}(j,i)$). Therefore, iteration $i$ only needs to compute interactions with particles with index $j$, where $i<j$. As a result, the work done by the algorithm is $N^2/2$ rather than $N^2$.

In this problem, assume the code is run on a dual-core processor, with infinite memory bandwidth. The processor implements invalidation-based cache coherence across the cores. The cache line size is 64 bytes.

```c
struct Particle {
    float force; // for simplicity, assume force is represented as a single float
};

Particle particles[N];

void compute_forces(int threadId) {
    // thread 0 takes first half, thread 1 takes second half
    int start = threadId * N/2;
    int end = start + N/2;
    for (int i=start; i<end; i++) {
        // only compute forces for each pair (i,j) once, then accumulate force
        // into *both* particle i and j
        for (int j=i+1; j<N; j++) {
            float force = gravity(i, j);
            particles[i] += force;
            particles[j] += force;
        }
    }
}
```

The question is on the next page.
A. (4 pts) The function `compute_forces` above is run by two threads on a dual-core processor. There is a correctness problem with the code. Using only the synchronization primitive:

\[ \text{atomicAdd}(\text{float*} \text{ addr, float val}) \]

Fix the correctness bug in the code. **However, to get full credit your solution should be efficient—it should do better than making \( N^2 \) calls to `atomic_add` (at least by an integer constant factor). Solutions that incur significant storage overhead or increase the amount of work done by the algorithm are not allowed.**

B. (2 pts) There is also a significant **performance problem** in the implementation that results in a speedup that is significantly lower than 2\( \times \) on the two-core processor. What is the problem?
C. (4 pts) Give an implementation of `compute_forces` that (1) achieves good workload balance between the two threads (2) does not significantly increase the amount of work performed (work should be no more than $N^2/2 + O(N)$) and (3) does not use fine-grained `atomicAdd` synchronization. However, you are allowed to allocate $O(N)$ storage and use a barrier. Pseudocode is fine.